Relational Incentive Contracts

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MOTIVATION AND MAIN RESULTS

- Study repeated interaction between a principal and an agent with:
  - i- Moral Hazard.
  - ii- Adverse Selection.
  - iii- Self-enforcement.
- Main Results:
  - i- The optimal contract is stationary (easy to characterize!!!!).
  - ii- The optimal contract is given by the static solution of moral hazard or adverse selection with endogenous limited liability.

PREFERENCES

- Time is discrete: $t = 0, 1, 2, \ldots$
- Two **risk neutral** agents: a principal and an agent.
- Preferences:

  Principal: $\pi_t = (1-\delta)\mathbb{E}[\sum_{\tau=t}^{\infty} \delta^{\tau-t}(y_\tau - W_\tau)]$

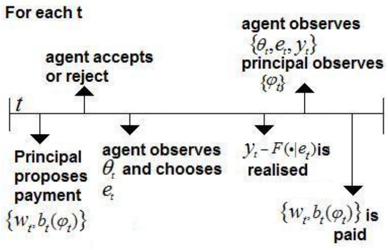
  Agent: $u_t = (1-\delta)\mathbb{E}[\sum_{\tau=t}^{\infty} \delta^{\tau-t}(W_t - c(e_t, \theta_t))]$

  Where y_t =output; W_t =total payments; $e_t =$ agent effort;$\theta_t =$ cost parameters in period t.
- Exogenous opportunity cost: $\bar{\pi}$ for the principal and $\bar{u}$ for the agent.

Time inside period

- Total payment: $W_t = w_t + b_t(\varphi_t)$.
- Agent's Information Set: $\{\theta_t, e_t, y_t\}$.
- Principal's Information Set: $y_t \in \varphi_t \subset \{\theta_t, e_t, y_t\}$.
- output $y_t \sim F(\cdot|e)$

Figure: Time Line



TWO PROBLEMS

- There is a **commitment technology for** w_t.
- There isn't a **commitment technology for** $b_t(\varphi_t)$.

There are two general frameworks:

Adverse selection and self-enforcement : The information set for the principal is $\{e_t, y_t\}$, but **not** $\{\theta_t\}$. Both players can refuse to pay $\{b_t(\varphi_t)\}$.

Moral Hazard and self-enforcement : The information set for the principal is $\{\theta_t, y_t\}$, but **not** $\{e_t\}$. Both players can refuse to pay $\{b_t(\varphi_t)\}$.

- $\theta_t \sim^{i.i.d.} P(\theta)$.
- $c(0,\theta) = 0$, $c'_e(e,\theta), c''_{e,e}(e,\theta) > 0$, for all θ
- $\max_e S(e,\theta) > \bar{u} + \bar{\pi} \geq S(0,\theta)$, for all θ. $S(e,\theta) = \mathbb{E}_y[y|e] - c(e,\theta)$.
- The information structure of the principal is fixed, i.e. for all t $\varphi_t = \{y_t, \theta_t\}$ or $\varphi_t = \{y_t, e_t\}$ or $\varphi_t = \{y_t\}$.

Contract

- $\varphi^t := (\varphi_0, ..., \varphi_{t-1}, \varphi_t)$.

Definition

A contract is a sequence of functions $\{f_t\}_t$ with $f_t(\varphi^{t-1}) = (w_t, b_t(\varphi_t), e_t(\theta_t))$.

The utilities are

$$u_t(\varphi^{t-1}) := \mathbb{E}_{\varphi_t}[(1-\delta)[W_t(\varphi_t) - c(e_t, \theta_t)] + \delta u_{t+1}(\varphi^t)]$$
$$\pi_t(\varphi^{t-1}) := \mathbb{E}_{\varphi_t}[(1-\delta)[y_t - W_t(\varphi_t)] + \delta \pi_{t+1}(\varphi^t)]$$

Self-Enforcing Contract

Definition

A self-enforcing contract is a contract s.t for $t \geq 0$ and all φ^t

$$\pi_t(\varphi^{t-1}) \geq \bar{\pi} \qquad [PC - P]$$

$$u_t(\varphi^{t-1}) \geq \bar{u} \qquad [PC - P]$$

$$e_t^{\varphi^{t-1}}(\theta_t) \in \arg\max_e \{w_t(\varphi^{t-1}) - c(e,\theta) + \dots$$

$$\dots + \mathbb{E}_y[b_t(\varphi^t) - u_{t+1}(\varphi^t)|e]\} \qquad [IC]$$

$$b_t(\varphi^t)(1-\delta) + \delta u_{t+1}(\varphi^t) \geq \delta \bar{u} \qquad [SE - A]$$

$$-b_t(\varphi^t)(1-\delta) + \delta u_{t+1}(\varphi^t) \geq \delta \bar{\pi} \qquad [SE - P]$$

A self-enforcing contract is optimal if there is no other self-enforcing contract with higher expected surplus $(\pi_t + u_t)$

Optimal Contract

Let $F(U) : [\bar{u}; C - \bar{\pi}] \to \mathbb{R}$ be the optimal value of a self-enforcing contract for the principal when the agent receives a value U.

$$F(U) = \max_{w,b(\varphi),e(\theta),U'(\varphi)} \{\mathbb{E}_{\theta,y}[(1-\delta)(y-w-b(\varphi))+\delta F(U'(\varphi))|e(\theta)]\}$$

s.t.

$$F(U'(\varphi)) \geq \bar{\pi} \qquad [PC - A]$$

$$U'(\varphi) \geq \bar{u} \qquad [PC - P]$$

$$U = \mathbb{E}_{\theta,y}[(1-\delta)(w + b(\varphi) - c(e,\theta)) + \delta U'(\varphi)|e(\theta)] \qquad [PK]$$

$$e(\theta) = \arg\max_{e}\{\mathbb{E}_y[(1-\delta)(b(\varphi) + \delta U'(\varphi))|e(\theta)] - c(e,\theta)\}[IC]$$

$$\delta\bar{u} \leq (1-\delta)b(\varphi) + \delta U'(\varphi) \qquad [SE - A]$$

$$\delta\bar{\pi} \leq -(1-\delta)b(\varphi) + \delta F(U'(\varphi)) \qquad [SE - P]$$

Optimal Contract

$F'(U) = -\lambda$, where λ is the shadow price of [PK] and $\lambda = 1$ (f.o.c. of w)

Lemma

F(U)=-U+C

Important assumptions:

1. Risk-Neutrality.
2. Perfect enforceable technology for w.
3. No IC for the principal.

Distribution doesn't affect incentives!!!!!! (F(U)+U=C)

With the previous lemma

$$F(U) = \max_{w,b(\varphi),e(\theta),U'(\varphi)} \{\mathbb{E}_{\theta,y}[(1-\delta)(y-w)-[(1-\delta)b(\varphi)+\delta U'(\varphi)]+\delta C|e(\theta$$

s.t.

$$C - \bar{\pi} \geq U'(\varphi) \geq \bar{u} \qquad [PC]$$

$$U = (1-\delta)(w - c(e,\theta)) + \mathbb{E}_{\theta,y}[(1-\delta)b(\varphi) + \delta U'(\varphi)|e(\theta)][PK]$$

$$e(\theta) = \arg\max_{e}\{\mathbb{E}_y[(1-\delta)b(\varphi) + \delta U'(\varphi)|e(\theta)] - c(e,\theta)\}[IC]$$

$$\delta\bar{u} \leq (1-\delta)b(\varphi) + \delta U'(\varphi) \leq \delta(C - \bar{\pi}) \qquad [SE]$$

Remark

b and U are perfect substitutes.

Proposition

Let $\{w, b(\varphi), e(\theta), U(\varphi)\}(U)$ be an optimal solution for $U \in [\bar{u}; C - \bar{\pi}]$. Then

$$\{w, b(\varphi) + \frac{\delta}{1-\delta}[U(\varphi) - U], e(\theta), U\}(U) \tag{1}$$

is a solution, i.e. the contract is stationary.

The full static problem is the **MAXIMUM C** s.t.

$$C = \max_{b(\varphi), e(\theta)} \{\mathbb{E}_{\theta,y}[y|e(\theta)] - c(e,\theta)\}$$

s.t.

$$e(\theta) \in \arg\max_{e} \{\mathbb{E}_y[(1-\delta)b(\varphi)|e(\theta)] - c(e,\theta)\}[IC]$$

$$\frac{\delta}{1-\delta}(C - \bar{u} - \bar{\pi}) \geq \sup b(\varphi) - \inf b(\varphi) \qquad [DE]$$

Proposition

If $F(y|e)$ satisfies the monotone likelihood ratio and $F(y|e = c^{-1}(x, \theta))$ is convex in x for all θ, then C solves

$$C = \max_{b(\theta,y),e(\theta)} \underbrace{\int_{\underline{\theta}}^{\bar{\theta}} \{\mathbb{E}[y|e(\theta)] - c(e(\theta), \theta)\} dP(\theta)}_{=K(e,\theta)}$$

s.t.

$$\frac{\delta}{1-\delta}(K(e,\theta) - (\bar{\pi} + \bar{u})) \geq \sup b(\theta, y) - \inf b(\theta, y)$$

$$\mathbb{E}_y[b(\varphi)L(y, e)|e] = c'_e(e, \theta)$$

Where $L(y, e) = \frac{f'_e(y|e)}{f(y|e)}$.

- **First Best:** $\mathbb{E}_y[yL(y, e(\theta))|e(\theta)] = c'_e(e^{FB}(\theta), \theta)$.
- Let $b^*(\varphi) \implies e(\theta) < e^{FB}(\theta)$.

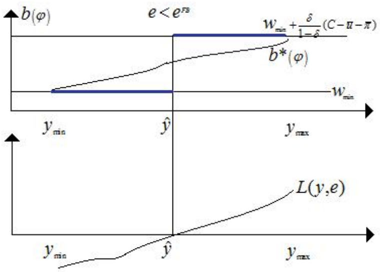


Figure: Optimal wage contract to implement e^*

$\uparrow \mathbb{E}_y[b(\varphi)L(y, e)|e] \rightarrow \uparrow c'_e(e) \rightarrow \uparrow e$

Assume that $c_\theta(e, \theta) = 0$ and

$$b(y) = \begin{cases} \underline{w} & \text{if } y < \hat{y} \\ \bar{w} & \text{if } y \geq \hat{y} \end{cases} \tag{2}$$

Then from the first order condition of IC

$$(\bar{w} - \underline{w})\mathbb{E}_y[L(y, e)I(y > \hat{y})|e] = c'(e)$$

Proposition

An optimal contract implements $e \leq e^{FB}$. If $e < e^{FB}$, then $b(y)$ satisfies (2), with $L(\hat{y}, e) = 0$ and $\bar{w} = \underline{w} + \frac{\delta}{1-\delta}(C - \bar{p}i - \bar{u})$.

- **Single Cross Property:** $c''_{e,\theta}(e,\theta) > 0$.
- $c'_{\theta}(e,\theta) > 0$.
- $c'''_{\theta,e^2}(e,\theta); c'''_{\theta,e,\theta}(e,\theta) \geq 0$.
- P is concave.
- $S(e,\theta)$ is differentiable and concave in e.

Proposition

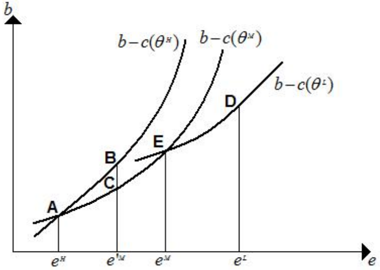
Suppose that $\varphi = \{e, y\}$, *then* C *solves*

$$C = \max_{e(\theta)} \underbrace{\int_{\underline{\theta}}^{\bar{\theta}} \{\mathbb{E}[y|e(\theta)] - c(e(\theta), \theta)\} dP(\theta)}_{=K(e,\theta)}$$

s.t. $e(\theta)$ *nonincreasing and*

$$\frac{\delta}{1-\delta}(K(e,\theta) - (\bar{\pi} + \bar{u})) \geq c(e(\underline{\theta}), \underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} c_\theta(e(x), x) dx [IC - DE]$$

Intuition



Since $b'(e(\theta)) = c_e'(e(\theta), \theta)$

If $e^M - e^H = dx$

$$\begin{aligned} dw &= c(e^M, \theta^M) - c(e^H, \theta^M) \\ &= c_e'(e(\theta_M), \theta_M)dx \end{aligned}$$

$$b(e(\theta)) = H + c(e(\theta), \theta) + \int_{\theta}^{\bar{\theta}} c_\theta'(e(x), x)dx$$

From the first order conditions

$$\underbrace{S'_e(e(\theta),\theta)p(\theta)[1+\mu\frac{\delta}{1-\delta}]}_{\text{Marginal benefit of increase effort}} = \underbrace{\mu c''_{e,\theta}(e(\theta),\theta)}_{\text{Direct effect of IC-DE}} + \underbrace{K(\theta)}_{\text{decreasing e}(\theta)\text{ constraint}}$$

with $e'(\underline{\theta}) = 0$ if $\mu > 0$.

- If $IC - DE$ is not active, $\mu = 0$
- If $e'(\theta) < 0$, then $K(\theta) = 0$
- If $e'(\theta) = 0$, then $K(\theta) > 0$.

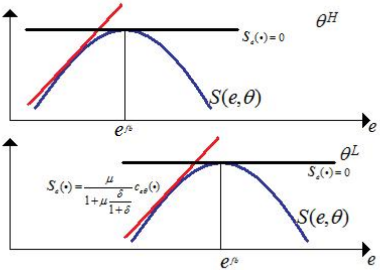


Figure: Less optimal effort with $\mu > 0$

Assume that $\mu > 0$. Two important conclusions:

All types are distorted from the first best

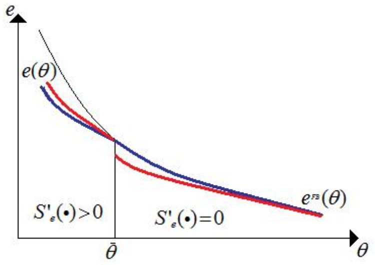


Figure: Optimal Contract effort

In the optimal contract, effort is constant for low types.

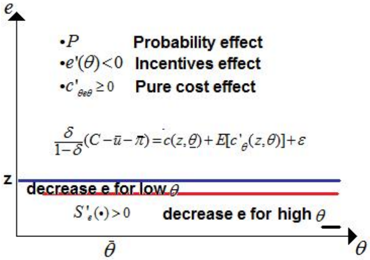


Figure: Decreaser effort is more cheaper for high θ

Proposition

If production is possible under a contract with adverse selection, the optimal effort schedule $e(\theta)$ takes one of the three forms:

1. *Pooling: $e(\theta)$ is the same for all cost types.*
2. *Partial Pooling: $e(\theta)$ is constant on $[\underline{\theta}, \hat{\theta}]$ and strictly decreasing on $(\hat{\theta}, \overline{\theta}]$.*
3. *First-Best: $e(\theta) = e^{FB}(\theta)$, where $e^{FB}(\theta)$ solves the problem without $IC - DE$ constraint.*

In either second-best scenario, $e(\theta) < e^{FB}(\theta)$.

- A simple framework to analyze incentives. Main features:
  - Risk-neutral agents.
  - Element to transfer utilities with perfect commitment.
- A model with endogenous liability. The size of the surplus will affect incentives.