A Positive Theory of Government Debt

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Introduction

Question:

- What determines the level of government debt?
    - Bohn (1998)

The paper studies how the steady-state level of debt is determined using a cash-in-advance model with nominal bonds.

- a benchmark model.
- an extended model.
- an extended model with stochastic gov't expenditure.

A Benchmark Model: Environment

- Discrete-time and infinite-horizon.
- A representative household:
  - $\sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$, $u(c_t, l_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \alpha l_t$ with $\alpha > 1$.
  - A worker and a shopper
  - Worker: Linear production technology: $n_t = y_t$, $(l_t + n_t = 1)$
  - Shopper: Cash-in-advance constraint: $\bar{p}_t c_t \leq \bar{m}_t$
  - Household budget constraint: $\bar{m}_{t+1} + q_t \bar{b}_{t+1} \leq \bar{p}_t n_t + (\bar{m}_t - \bar{p}_t c_t) + \bar{b}_t$
- Government:
  - Government budget constraint: $\bar{p}_t g + \bar{B}_t \leq (\bar{M}_{t+1} - \bar{M}_t) + q_t \bar{B}_{t+1}$
- Notation: $\mu_t = \bar{M}_{t+1}/\bar{M}_t - 1$, $p_t = \bar{p}_t/\bar{M}_t$, $m_t = \bar{m}_t/\bar{M}_t$, $b_t = \bar{b}_t/\bar{M}_t$, $B_t = \bar{B}_t/\bar{M}_t$.

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A Benchmark Model: Household's Problem

Given gov't debt policy $B' = \mathcal{B}(B)$ and associated μ satisfying the GBC, the household's problem can be written as

$$V(m, b, B) = \max_{\{c,n,m',b'\}} u(c, 1-n) + \beta V(m', b', B')$$

s.t.

$$HBC : (1+\mu)(m' + qb') = pn + (m - pc) + b$$

$$CA : pc \leq m$$

A Benchmark Model: Competitive Equilibrium

Market Clearing:

- $c + g = y$
- $m' = M' = 1$
- $b' = B'$

FONCs and Market Clearing imply

- $q = \alpha / u'_c$
- $qp'/p(1+\mu) = \beta$ (or $q\bar{p}'/\bar{p} = \beta$)
- $pc = 1$

A Benchmark Model: A Markov-Perfect Equilibrium

A Markov-Perfect Equilibrium is a set of functions $[\mathcal{V}, \mathcal{B}, \mathcal{C}]$ such that $c = \mathcal{C}(B)$ and $B' = \mathcal{B}(B)$ solve

$$\mathcal{V}(B) = \max_{\{c,B'\}} u(c, 1 - c - g) + \beta \mathcal{V}(B')$$

s.t.

$$\frac{\beta}{\alpha}\mathcal{C}(B')u_c(\mathcal{C}(B')) + \beta B'\mathcal{C}(B') = c + cB + g$$
$$u_c/\alpha \geq 1$$

A Benchmark Model: Analytical Results

FONC: $\frac{\frac{u_c(c')}{\alpha}-1}{1+B'} = \frac{\frac{u_c(c)}{\alpha}-1}{1+B}\left[1+\frac{B\mathcal{C}_B(B')}{\mathcal{C}(B)}+\frac{u_c(\mathcal{C}_B(B'))\mathcal{C}_B(B')}{\alpha\mathcal{C}(B')}(1-\sigma)\right]$

- P1: There are two steady states (B^* and B^{**}); B^{**} is first-best.
- P2: B^{**} is unstable
- P3: $B^* > 0$ iff $\sigma > 1$, $B^* = 0$ iff $\sigma = 1$, and $B^* < 0$ iff $\sigma < 1$
- P4: B^* is increasing in σ

A Benchmark Model: An Example

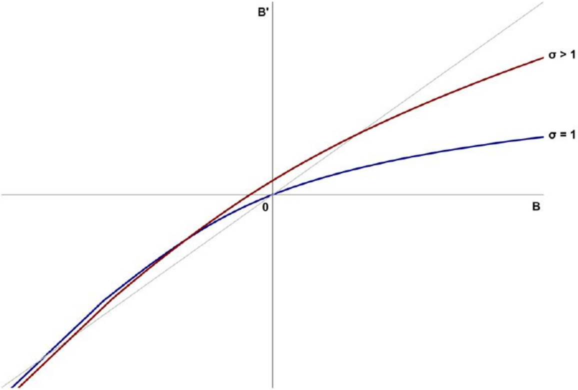

Fig. 1. Debt functions for $u(c,\ell) = \frac{c^{1-\sigma}-1}{1-\sigma} + \frac{\ell^{1-\varsigma}}{1-\varsigma}$, $\varsigma > 1$, $g > 0$.

A Benchmark Model: Characterizing B^*

Two opposing forces:

- An increase in $\bar{p}_t$ lowers the real value of outstanding nominal debt.
- But, it distorts the household decision due to cash-in-advance constraint $(\bar{p}_t c_t \leq \bar{m}_t)$
  - The distortion depends on the household's willingness to substitute away from c_t.
  - High σ = Low IES

An Extended Model: Environment

- $c_{1,t}$ (cash-goods) and $c_{2,t}$ (credit-good)
- $c_{1,t} + c_{2,t} + g = y_t$
- $\bar{p}_t c_{1,t} \leq \bar{m}_t$
- $u(c_1, c_2, l) = \frac{1}{1-\sigma}\left[\left[(\alpha c_1^\rho + (1-\alpha) c_2^\rho)^{\frac{\gamma}{\rho}} l^{1-\gamma}\right]^{1-\sigma} - 1\right]$
- HBC: $\bar{p}_t c_{2,t} + \bar{m}_{t+1} + q_t \bar{b}_{t+1} = (1-\tau_t) n_t + (\bar{m}_t - \bar{p}_t c_{1,t}) + \bar{b}_t$
- GBC: $\bar{M}_t + \bar{B}_t + \bar{p}_t g = \tau_t n_t + \bar{M}_{t+1} + q_t \bar{B}_{t+1}$

An Extended Model: Calibration

Table: Parameter Values

Parameter	α	β	γ	ρ	σ	g
Values	0.0237	0.9774	0.303	-2.800	4.250	0.054

Table: Target

Variables	Data (average over 1962-2006 in the U.S.)
$B(1+\mu)/py$	0.31
c_1/c_2	0.37
g/y	0.18
n	0.3
π	1.039
r	1.023

An Extended Model: Comparative Statics

Table: Comparative Statics

	Benchmark	Lower σ	Lower ρ	Lower α
$B(1+\mu)/py$	0.310	0.294	0.436	0.278

*Recall $u(c_1, c_2, l) = \frac{1}{1-\sigma}\left[\left[(\alpha c_1^{\rho} + (1-\alpha)c_2^{\rho})^{\frac{\gamma}{\rho}} l^{1-\gamma}\right]^{1-\sigma} - 1\right]$

- More substitutable $c_{1,t}$ => Less distortion from inflating debt => Lower debt

An Extended Model With Stochastic g_t

- $0 < g_L < g_H$
- $\text{Prob}(g_{t+1} = g_L | g_t = g_L) = \theta_L$
- $\text{Prob}(g_{t+1} = g_H | g_t = g_H) = \theta_H$

An Extended Model With Stochastic g_t: Calibration

Table: Calibration for Civil War and WWI (based on U.S. data from 1832-1929)

Parameter	α	β	γ	ρ	σ	g_L	g_H	θ_L	θ_H
Values	0.315	0.971	0.303	-0.5	4.25	0.006	0.031	0.978	0.778

Table: Calibration for WWII (based on U.S. data from 1930-1941)

Parameter	α	β	γ	ρ	σ	g_L	g_H	θ_L	θ_H
Values	0.056	0.971	0.303	-4.5	4.25	0.022	0.096	0.978	0.778

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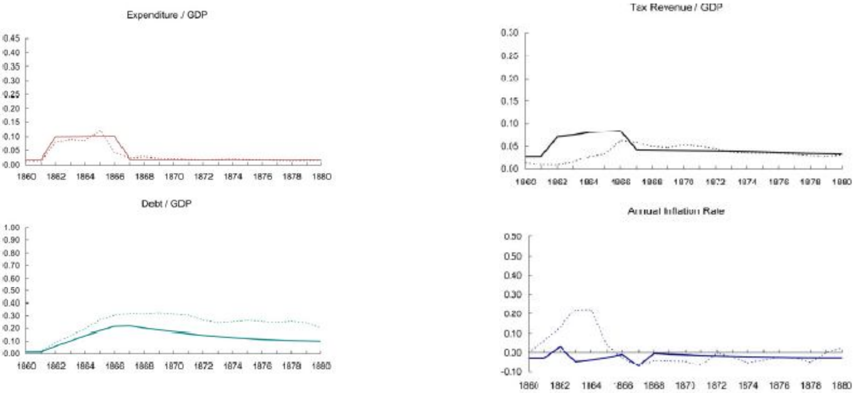
An Extended Model With Stochastic g_t: Civil War



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