Endogenous Incomplete Markets, Enforcement Constraints, and Intermediation

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Introduction

This paper studies the decentralization of the constrained efficient allocation of an economy with

- incomplete risk sharing due to limited enforcement
- endogenous capital accumulation

Key: the value from financial Autarky is *endogenous*

Environment

- Discrete time and infinite horizon.
- Aggregate uncertainty and idiosyncratic shock at t, s^t, with transition law $\pi(s^r|s^t)$.
- One consumption good and one capital good, perfectly transferrable, capital depreciation rate δ
- Production: $y(s^t) = f(z(s^t), K(s^{t-1}), L(s^t))$, concave in K, $f_{LK} > 0$, and homogeneous of degree one.
  $F(s^t) = (1-\delta)K(s^{t-1}) + f(s^t)$

Environment - cont'd

- Two types of consumers, a continuum for each type, $I = \{1, 2\}$, each type has population 1
  - Preference: $\sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t u(c_i(s^t))$
    $u(\cdot)$ smooth, concave, unbounded below, $u'(0^+) = \infty$, $u'(\infty) = 0$.
  - Endowment: $\varepsilon_i(s^t)$
  - State variables for recursive problem:
    $S_i(s^t) = \left(\{\varepsilon_j(s^t)\}_{j\in I}, z(s^t), K(s^{t-1})\right)$

Social Planner's problem

$$
\begin{array}{ll}
\max_{\{c_i\}_{i\in I},K} \sum_{i\in I} \alpha_i \sum_{t=0}^{\infty} \sum_{s^t} \pi(s^t)\beta^t u(c_i(s^t)) & \text{s.t.} \\
\sum_{i\in I} c_i(s^t) + K(s^t) = F(s^t) & \text{feasibility} \\
\sum_{r=t}^{\infty} \sum_{s^r} \beta^{r-t}\pi(s^r|s^t)u(c_i(s^r)) \geq V(S_i(s^t)) & \text{participation}, \gamma_i(s^t)
\end{array}
$$

- α_i: social planner's weight at time 0
- $V(S_i(s^t))$ financial Autarky, with access to labor income $V(S_i(s^t)) = \sum_{r=t}^{\infty} \sum_{s^r|s^t} \beta^{r-t}\pi(s^t|s^t)u(f_L(s^r)\varepsilon_i(s^r))$
- Comment:
  - $V(S_i(s^t))$ endogenous
  - convexity of the set of constraints
  - computational difficulty

Intratemporal FOC

FOC wrt $\{c_i(s^t)\} \Rightarrow$

$$\frac{u'(c_i(s^t))}{u'(c_{-i}(s^t))} = \lambda(s^t)$$

where $\lambda(s^t) = \frac{\alpha_{-i}(s^t)}{\alpha_i(s^t)}$ and $\alpha_i(s^t) = \alpha_i + \sum_{r=0}^{t} \gamma_i(s^r)$.

- $\lambda(s^t)$: the ratio of social planner's weight at s^t when the problem is recursively formulated
- social planner's weights change only when enforcement constraints bind

Intertemporal FOC

FOC wrt $K(s^t) \Rightarrow$

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \frac{u'(c_i(s^{t+1}))}{u'(c_i(s^t))} (1 + \nu_i(s^{t+1})) F_K(s^{t+1}) \right\}$$

$$-\beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{\nu_j(s^{t+1})}{u'(c_j(s^t))} V_k(S_j(s^{t+1})) \right\}, \quad i \in I$$

- $\nu_i(s^{t+1}) = \frac{\alpha_i(s^{t+1}) - \alpha_i(s^t)}{\alpha_i(s^t)} = \frac{\gamma_i(s^{t+1})}{\alpha_i(s^t)}$: growth rate of social planner's Pareto weight
- $\nu_i(s^{t+1}) > 0$ iff the enforcement constraint for consumer i is binding.
- if $\nu_i(s^{t+1}) = 0, \forall i$, FOC for an unconstrained economy.

Intertemporal FOC: Analysis

$$\ldots = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \frac{u'(c_i(s^{t+1}))}{u'(c_i(s^t))} (1 + \nu_i(s^{t+1})) F_K(s^{t+1}) \right\} \ldots$$

- Binding enforcement constraint
  - increases the planner's marginal rate of substitution.
  - makes the marginal rate of substitution differ across agents
- for all i, $\max_{j=1,2} \frac{u'(c_j(s^t))}{u'(c_j(s^{t-1}))} = \frac{u'(c_i(s^{t+1}))}{u'(c_i(s^t))}(1 + \nu_i(s^{t+1}))$
- Needs *type specific* constraints for decentralization (Alvarez and Jermann (2000))

Intertemporal FOC: Analysis

$$\ldots - \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{\nu_j(s^{t+1})}{u'(c_j(s^t))} V_k(S_j(s^{t+1})) \right\}, \forall i$$

- Binding enforcement constraint leads to lower capital growth
  - decreases the planner's marginal rate of substitution
  - decreases the marginal rate of substitution for all agents
- Trade-off between insurance and growth
- Needs an *economy-wide* constraint for decentralization
- The effect of capital on utility from Autarky is
  - persistent
  - economy-wide (externality)

Intermediary

- An intermediary at period t:
  - lives for two periods, payout dividends $d(s^{t+1}|s^t)$ at the second period
  - collects contingent savings from HH and sell loans to them
  - allowed to set state-contingent borrowing constraints on HH
  - transforms net savings into capital and rents it to the production sector
  - is a price taker
- Intermediaries' problem:

$$\max_{k(s^t)} -k(s^t) + \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)\left[r(s^{t+1} + 1 - \delta)k(s^t)\right] \quad \text{s.t.}$$

$$k(s^t) \leq B(s^t) \qquad (\psi(s^t))$$

Household's problem

- State variables: wealth and exogenous factors
  $\omega(s^t) = a_i(s^t|s^{t-1}) + \theta_i(s^{t-1})d(s^t) + \varepsilon_i(s^t)w(s^t)$
- Choice variables: $x(s^t) = \{c(s^t), \{a(s^{t+1}|s^t)\}_{s^{t+1}}, \theta(s^t)\}$

$$
\begin{aligned}
W_i(s^t) = \quad & \max_x(s^t) u(c_i(s^t)) + \beta \textstyle\sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) W_i(s^{t+1}) \\
\text{s.t.} \quad & c_i(s^t) + \textstyle\sum_{s^{t+1}} a_i(s^{t+1}|s^t) q(s^{t+1}|s^t) + p_i(s^t)\theta(s^t) \leq \omega(s^t) \\
& a_i(s^{s+1}|s^t) + \theta(s^{t+1}|s^t) d(s^{t+1}|s^t) \geq A_i(s^{t+1}|s^t), \forall s^{t+1}|s^t
\end{aligned}
$$

Definition of Competitive Equilibrium

Definition

A *Competitive Equilibrium* with initial asset holding $\{\omega_i(s^0)\}_i$ and $K(s^{-1})$ is prices $\{w(s^t), r(s^t), q(s^{t+1}|s^t), p(s^t)\}$, constraints $\{A_i(s^{t+1}|s^t), B(s^{t+1}|s^t)\}$ and allocations $\{k(s^t), K(s^t), c_i(s^t), a_i(s^{t+1}|s^t), \theta_i(s^t)\}$such that

- given prices, allocations solve problems of banks, firms and consumers.
- A-D security markets clear: $\sum_i a_i(s^{t+1}|s^t) = F_k(s^{t+1}|s^t) - d(s^{t+1}|s^t), \forall s^{t+1}$
- the capital market clears: $k(s^t) = K(s^t), \forall s^t$
- stock market clears:$\theta_1(s^t) + \theta_2(s^t) = 1, \forall s^t$.
- the product market and the labor market at any s^t clear

Proof Sketch

- choose $\{\alpha_i\}$ arbitrarily, solve for the constrained efficient allocation and lagrangian multipliers. Search for $\{\alpha_i\}$ that matches the initial wealth distribution and the initial capital.
- Choose prices and dividends that make compatible FOC's of the planner and FOC's of the firms', consumers' and banks' problems.
- backward engineer constraints $\{A_i\}$ and B from where prices and allocations imply binding constraints.
- More on step 2

Proof Sketch - cont'd

The planner's FOC:

$$1 = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \frac{u'(c_i(s^{t+1}))}{u'(c_i(s^t))} (1+\nu_i(s^{t+1})) F_K(s^{t+1}) \right\}$$
$$-\beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{\nu_j(s^{t+1})}{u'(c_j(s^t))} V_k(S_j(s^{t+1})) \right\}, \quad i \in I$$

HH's FOC:

$$q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t) \frac{u'(c_i(s^{t+1}))}{u'(c_i(s^{t+}))} (1+\gamma_i(s^{t+1}))$$

Let $\gamma_i(s^{t+1}) = \nu_i(s^{t+1})$.
Firm's FOC:

$$r(s^{t+1}) + 1 - \delta = F_k(s^{t+1})$$

Proof Sketch - cont'd

The planner's FOC:

$$1 = \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)[r(s^{t+1}) + 1 - \delta]$$

$$-\beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{\nu_j(s^{t+1})}{u'(c_j(s^t))} V_k(S_j(s^{t+1})) \right\}, \quad i \in I$$

Bank's FOC:

$$1 = \sum_{s^{t+1}|s^t} q(s^{t+1}|s^t)[r(s^{t+1}) + 1 - \delta] - \psi(s^t) \Rightarrow$$

$$\psi(s^t) = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t) \left\{ \sum_{j=1,2} \frac{\nu_j(s^{t+1})}{u'(c_j(s^t))} V_k(S_j(s^{t+1})) \right\}$$

$$d(s^{t+1}|s^t) = \frac{\nu_j(s^{t+1})}{u'(c_i(s^{t+1}))} V_k(S_j(s^{t+1})$$