Equilibrium Selection in global games with strategic complementarities

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November 2010

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- We refer to the above game $G(v)$ as a global game.

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- Focus on the games with strategic complementarities;

Assumptions

1. $(A1)$ Strategic Complementarities:

   If $a_i \geqslant a_i'$ and $a_{-i} \geqslant a_{-i}'$ then $\forall \theta$, $\Delta u_i\left(a_i, a_i', a_{-i}, \theta\right) \geqslant \Delta u_i\left(a_i, a_i', a_{-i}', \theta\right)$

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2. $(A2)$ Dominance regions

$$\begin{aligned} \exists \underline{\theta} \text{ such that } \forall \theta &< \underline{\theta} \text{ and } \forall a'_i \neq 0,\ \Delta u \left(0, a'_i, a_{-i}, \theta\right) > 0 \\ \exists \overline{\theta} \text{ such that } \forall \theta &> \overline{\theta} \text{ and } \forall a'_i \neq 1,\ \Delta u \left(1, a'_i, a_{-i}, \theta\right) >, 0\ \overline{\theta} > \underline{\theta} \end{aligned}$$

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3. $(A3)$ State monotonicity:

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   $$\Delta u\left(a_i, a'_i, a_{-i}, \theta\right) - \Delta u\left(a_i, a'_i, a_{-i}, \theta'\right) \geqslant K_0\left(a_i - a'_i\right)\left(\theta - \theta'\right)$$

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4. $(A4)$ Payoff continuity

$$u_i \left(a_i, a_{-i}, \theta\right) \text{ is continuous in all arguments}$$

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- A mixed strategy is a probability distribution over pure strategies

Limit Uniqueness

Theorem (1)

$\exists$ *an increasing strategy profile* s^* *such that if, for each* $v > 0$, s^v *is a strategy profile that survives iterative deletion of strategies in* $G(v)$, *then*

$$\lim_{v \longrightarrow 0} s_i^v(x_i) = s_i^*(x_i)$$

for almost all $x_{i.}$

This theorem states that as noise vanishes the iterative deletion of dominated strategies selects a unique Bayesian Nash Equilibrium of the game

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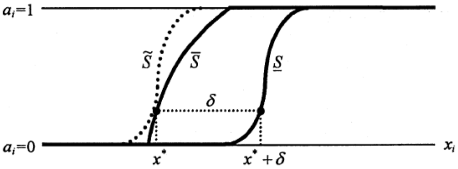
- 

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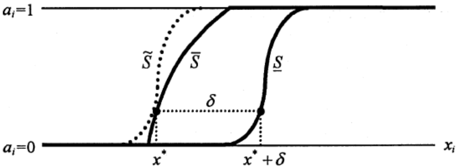
- 

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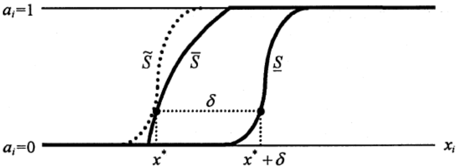
- 

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- $\Longrightarrow \delta = 0$ and so $\overline{S} = \underline{S} = \widetilde{S}$

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- More precisely, the set of rationalzable strategies of the simplified game and original game converges.
- This result holds for any prior and any finite number of players or conitnuum of players.

A partial characterization of equilibrium

We can characterize the surviving equilibria of the game when noise is small

Definition

$Q(\varepsilon, v)$ is the set of parameters θ for which no Nash Equilibrium action profile a of the complete information game with payoffs $\left(u_i\left(\cdot, \theta'\right)\right)_{i=1}^{I}$ for some $\theta' \in [\theta - \varepsilon, \theta + \varepsilon]$, such that for every strategy s^v surviving iterative deletion of dominated strategies in $G(v)$, $\forall i \quad \left|s_i^v(\theta) - a_i\right| < \varepsilon$

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- Define $Q(\varepsilon, v)$ be a set of θ for which surviving strategy profiles in $G(v)$ do not prescribe all players to play close to some pure strategy Nash equilibrium of a complete information game.

A partial characterization of equilibrium

Theorem (2)

In $G(v)$ in the limit as $v \longrightarrow 0$ for almost all payoff parameter θ, players play arbitrary close to some pure strategy Nash Equilibrium of the complete information game with payoffs $u(\cdot, \theta')$ that is arbitrary close to θ, i.e. $\forall \varepsilon > 0$ there is $\overline{v}$ s.t.$\forall v < \overline{v}$, $|\theta' - \theta| < \epsilon$

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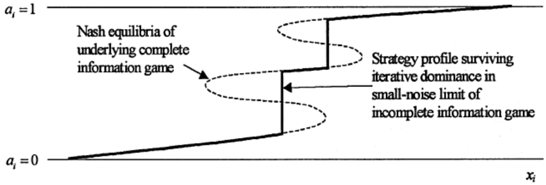
- 

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There exists a two-person, four-action game satisfying $(A1) - (A5)$ in which for different noise structure different equilibria are selected in the limit as the signal errors vanish.

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Theorem (4)

If the complete information game at some payoff parameter θ is quasiconcave in a_i and has local potential maximizer (LP-maximizer) a^* then $s^*(\theta) = a^*$ regardless of the noise structure

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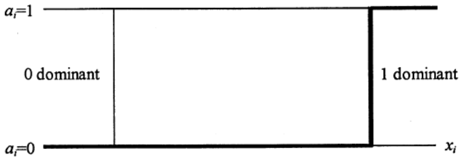
- 

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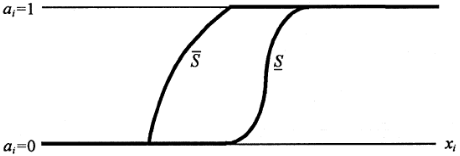
- 

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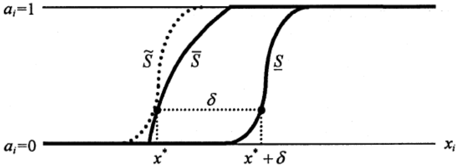
- 

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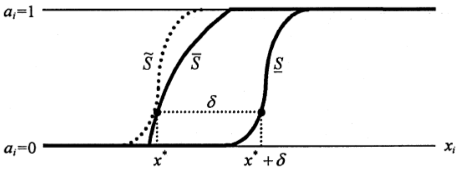
- 

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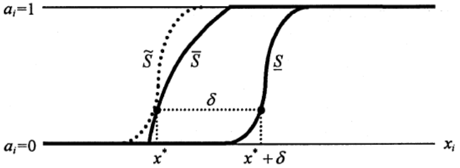
- 

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- Since $\widetilde{S} \geqslant \overline{S}$ it follows by $(A1)$ that $a^* \geqslant \overline{S}(x^*) = \underline{S}(x^* + \delta)$

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- That would lead to contradiction unless $\widetilde{S}(x) = \underline{S}(x)$

Appendix (Potential Games)

Definition (Potential Function)

Let N be a finite set of players, Y_i be the set of $i's$ strategies and $u_i : Y \longrightarrow R$ be $i's$ payoff function. Then a function $P : Y \longrightarrow R$ is called a potential function if for every i and every Y_{-i} we have

$$u(x, y_{-i}) - u\left(x', y_{-i}\right) = P\left(x, y_{-i}\right) - P\left(x', y_{-i}\right)$$

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- A potential function is a common payoff function such that a change is player $i's$ payoff from switching action to another is always the same as the change in the potential function.

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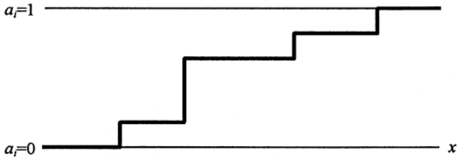
- 

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  - hence as noise vanishes, players must play potential maximizing action