Household Choices and Child Development

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Main Question

- What is the relationship between parental time allocation, household expenditure patterns and child quality outcomes?

- Estimate dynamic child quality production function, with (potentially) observed factors of production such as (active and passive) parental time and expenditures on children.
  - How do the factor loadings change as the child grows up?
  - Do different age children benefit more from different inputs?

- What is the effect of parental employment on child quality?
  - Comparative statics exercise with wage increase
MODEL

- Single children families: child’s age = t
- Household’s life begins with the birth of a child, infinitely lived.
- Child quality investment decisions at $t = 1, \ldots, T$
  - active ($\tau_{it}$, $i = 1, 2$) and passive ($z_{it}$, $i = 1, 2$) childcare time for each parent, and expenditures ($e_t$) on child.
- Household utility function:
  $$u(l_{1t}, l_{2t}, c_t, k_t) = \alpha_1 \ln l_{1t} + \alpha_2 \ln l_{2t} + \alpha_3 \ln c_t + \alpha_4 \ln k_t$$
  - where $l_{it}$, $i = 1, 2$ are leisure for mother and father, $c_t$ is consumption and $k_t$ is latent child quality at time $t$.
- Child quality production function:
  $$k_{t+1} = f(k_t, \tau_{1t}, \tau_{2t}, z_{1t}, z_{2t}, e_t) = R_t \tau_{1t}^{\delta_{1t}} \tau_{2t}^{\delta_{2t}} z_{1t}^{\delta_{3t}} z_{2t}^{\delta_{4t}} e_t^{\delta_{5t}} k_t^{\delta_{6t}}$$
**Dynamic Problem**

\[ V_t(S_t) = \max_{l_1t, \tau_1t, l_2t, \tau_2t, z_1t, z_2t, e_t} u(l_1t, l_2t, c_t, k_t) + \beta \mathbb{E}_t V_{t+1}(S_{t+1}) \]

s.t. \[ TT = l_{it} + h_{it} + \tau_{it} + z_{it}, \quad i = 1, 2 \]
\[ c_t + e_t = w_1t h_1t + w_2t h_2t + I_t \]

- where \( h_{it} \) denotes market hours, \( w_{it} \) denote wages, and \( I_t \) is non-labor income.
- The state vector \( S_t = (k_t, w_1t, w_2t, I_t) \)
- No Saving or Borrowing!
**Terminal Value**

- Let $\tilde{S}_t = (w_{1,t}, w_{2,t}, I_t)$
- Terminal level of child quality $\equiv k_{T+1}$

$$V_{T+1}(\tilde{S}_{T+1}; k_{T+1}) = V_{T+1}^{static}(\tilde{S}_{T+1}) + \psi \alpha_4 \ln k_{T+1}$$

- where

$$V_{T+1}^{static}(\tilde{S}_T) = \max_{l_{1T}, l_{2T}} \alpha_1 \ln l_{1T} + \alpha_2 \ln l_{2T}$$

$$+ \alpha_3 \ln [w_{1T}(TT - l_{1T}) + w_{2T}(TT - l_{2T}) + I_T]$$

$$\forall \tau = T + 1, T + 2, ...$$
Econometric Specification

- **Heterogeneity**: HH preference parameters $(\alpha_1 \alpha_2 \alpha_3)$ distributed $G(\alpha; \theta)$ in the population

- Child quality production function parameters

  $$\delta_{jt} = \exp(\gamma_{j0} + \gamma_{j1}t), \quad j = 1, \ldots, 6; \quad t = 1, \ldots, T$$

- Latent non-labor income: $I_t^* = \mu_I + \epsilon_{It}, \quad \epsilon_{It} \sim iid \ N(0, \sigma_I^2)$, thus

  $$I_t = \max(0, I_t^*), \quad \forall t$$

- Wage process:

  $$\ln w_{it} = \mu_{it} + \epsilon_{it}, \quad i = 1, 2$$

  $$\epsilon_t \sim iid \ N(0, \Sigma_w), \quad \epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$$

  $$\mu_{it} = \mu_i^0 + \mu_i^1edu_i + \mu_i^2age_{it} + \mu_i^3age_{it}^2, \quad i = 1, 2$$
Measuring Child Quality

- Child with latent quality $k_t$, with measured $k_t^*$ correct answers
- Probability of answering each question right on a test:

$$p(k_t) = \frac{k_t}{1 + k_t}$$

- $k_t^* \sim \text{Binomial}(\bar{k}, p(k_t))$, $\bar{k} =$ total number of questions

- $k_0$ not observed. Prior for $p_0 \sim \text{Beta}(1, 1)$ i.e. $U[0, 1]$
  - $k_0^*$ is observed, update $\Rightarrow$ posterior is $\text{Beta}(1 + k_0^*, (\bar{k} - k_0^*) + 1)$
  - Draw $\tilde{p}$ from that posterior $\Rightarrow \tilde{k} = \frac{\tilde{p}}{1 - \tilde{p}} \equiv k_0$
  - Construct $\{k_t\}$ sequence from $k_0$, using the HH inputs

- $k_{2002}^*$ is observed at age $t'$ for that child
- $k_{2002}^*$ drawn from $\text{Binomial}(\bar{k}, p(k_{t'}))$
DATA AND ESTIMATOR

▶ PSID-CDS 1997:
  ▶ 3500 children in 2400 HHs who are part of the PSID sample
  ▶ Data collected for up to 2 children < 13 years old, per HH
  ▶ Follow up in 2002-2003
  ▶ Time diaries and letter word test results
  ▶ Restrict to: HHs with both biological parents in both waves, and with only one child


▶ Estimate using Method of Simulated Moments
**Child Quality Technology Parameters**

Estimated Child Development Parameters by Child Age

- **Mother, Active**
- **Father, Active**
- **Mother, Passive**
- **Father, Passive**
- **Expenditures**

Input Productivity vs. Age

Student Version of MATLAB
**Comparative Statics: Wage Changes**

For each period $t$, increase the wage draws of mothers and fathers:

- **A 1% increase in the mother’s wage:**
  - Mother’s time with child ↓
  - Father’s time with child ↑
  - Expenditures ↑

  Measured and latent child quality ↑ by 0.0005 and 0.0049

- **A 1% increase in the father’s wage:**
  - Mother’s time with child ↑
  - Father’s time with child ↓
  - Expenditures ↑ (by more than previous case)

  Measured and latent child quality ↑ by 0.0007 and 0.0068
### APPENDIX

**Table 2: Parent’s Time with Child by Child Age**

<table>
<thead>
<tr>
<th>Child Age</th>
<th>Active Time (Avg.)</th>
<th>Passive Time (Avg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-5</td>
<td>32.65</td>
<td>16.40</td>
</tr>
<tr>
<td>6-8</td>
<td>20.86</td>
<td>12.65</td>
</tr>
<tr>
<td>9-11</td>
<td>19.45</td>
<td>13.83</td>
</tr>
<tr>
<td>12-15</td>
<td>16.12</td>
<td>17.99</td>
</tr>
</tbody>
</table>

### Table 3: Parent’s Labor Supply by Child Age

<table>
<thead>
<tr>
<th>Child Age</th>
<th>Fraction Working &gt; 0 Hours</th>
<th>Average Hours Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-5</td>
<td>0.831</td>
<td>30.67</td>
</tr>
<tr>
<td>6-8</td>
<td>0.829</td>
<td>35.91</td>
</tr>
<tr>
<td>9-11</td>
<td>0.862</td>
<td>36.63</td>
</tr>
<tr>
<td>12-15</td>
<td>0.794</td>
<td>34.67</td>
</tr>
</tbody>
</table>
Figure 1: Average Letter Word Score by Child Age

Mean Child Quality by Age

Notes: Sample of intact households (mother and father present in household) with one child.

### Table 4: Preference Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean of $\alpha_1$</td>
<td>0.130</td>
<td>(0.0067)</td>
</tr>
<tr>
<td>Mean of $\alpha_2$</td>
<td>0.169</td>
<td>(0.0050)</td>
</tr>
<tr>
<td>Mean of $\alpha_3$</td>
<td>0.329</td>
<td>(0.0101)</td>
</tr>
<tr>
<td>Mean of $\alpha_4$</td>
<td>0.373</td>
<td>(0.0054)</td>
</tr>
<tr>
<td>Std. of $\alpha_1$</td>
<td>0.104</td>
<td>(0.0105)</td>
</tr>
<tr>
<td>Std. of $\alpha_2$</td>
<td>0.081</td>
<td>(0.0044)</td>
</tr>
<tr>
<td>Std. of $\alpha_3$</td>
<td>0.092</td>
<td>(0.0074)</td>
</tr>
<tr>
<td>Std. of $\alpha_4$</td>
<td>0.205</td>
<td>(0.0101)</td>
</tr>
<tr>
<td>Correlation of $\alpha_1$ and $\alpha_2$</td>
<td>0.492</td>
<td>(0.109)</td>
</tr>
<tr>
<td>Correlation of $\alpha_1$ and $\alpha_3$</td>
<td>-0.254</td>
<td>(0.089)</td>
</tr>
<tr>
<td>Correlation of $\alpha_2$ and $\alpha_3$</td>
<td>0.403</td>
<td>(0.088)</td>
</tr>
<tr>
<td>$\psi$ (Terminal Payoff to Child Quality)</td>
<td>43.67</td>
<td>(29.13)</td>
</tr>
</tbody>
</table>
## APPENDIX

Table 9: **Comparative Statics**

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>Increase in Mother’s Wage</th>
<th>Increase in Father’s Wage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Measured Child Quality (Age 16)</td>
<td>51.99</td>
<td>0.0005</td>
<td>0.0007</td>
</tr>
<tr>
<td>Mean Latent Child Quality (Age 16)</td>
<td>11.15</td>
<td>0.0052</td>
<td>0.0068</td>
</tr>
<tr>
<td>Mean Hours Work (Mother)</td>
<td>33.02</td>
<td>1.052</td>
<td>-1.023</td>
</tr>
<tr>
<td>Mean Hours Work (Father)</td>
<td>43.32</td>
<td>-0.59</td>
<td>0.59</td>
</tr>
<tr>
<td>Mean Active Time w/ Child (Mother)</td>
<td>19.39</td>
<td>-0.49</td>
<td>0.49</td>
</tr>
<tr>
<td>Mean Active Time w/ Child (Father)</td>
<td>15.76</td>
<td>0.39</td>
<td>-0.39</td>
</tr>
<tr>
<td>Mean Passive Time w/ Child (Mother)</td>
<td>16.76</td>
<td>-0.49</td>
<td>0.49</td>
</tr>
<tr>
<td>Mean Passive Time w/ Child (Father)</td>
<td>15.43</td>
<td>0.39</td>
<td>-0.39</td>
</tr>
<tr>
<td>Mean Leisure (Mother)</td>
<td>42.84</td>
<td>-0.40</td>
<td>0.38</td>
</tr>
<tr>
<td>Mean Leisure (Father)</td>
<td>37.49</td>
<td>0.36</td>
<td>-0.36</td>
</tr>
<tr>
<td>Mean Child Expenditures</td>
<td>241.25</td>
<td>0.40</td>
<td>0.57</td>
</tr>
<tr>
<td>Mean Household Consumption</td>
<td>1,325</td>
<td>0.39</td>
<td>0.59</td>
</tr>
<tr>
<td>Mean Utility</td>
<td>–</td>
<td>0.13</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Notes: This table reports elasticity estimates from an increase in mother’s (2) and father’s wage offer (3). Mean Measured Child Quality (Age 16) is the average level of measured child quality (test score on scale of 0 to 57) at the end of age 16 or the start of period $t = 17, k_{17}^*$. Mean Latent Child Quality (Age 16) is the latent value of child quality at the end of age 16 or the start of period $t = 17, k_{17}$. 