The Dynamics of Productivity in the Telecommunications Equipment Industry

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Estimation of Cobb-Douglas Production

$$Y_{it} = A_{it} K_{it}^{\beta_k} L_{it}^{\beta_l}, \quad \text{taking logs}$$
$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \eta_{it}$$

Parameters for firm i in period t:

- β_0 is mean productivity
- ω_{it} idiosyncratic productivity observed by firm
- η_{it} iid measurement error not observed by firm/econometrician
- β_k and β_l are output elasticities
- ω_{it} and η_{it} mean 0

Problems with Naive OLS

If investment decision of a firm is dynamic and firm can exit:

- **Simultaneity Bias on** β_l: Firm chooses l_{it} after seeing ω_{it}
  - Positive correlation creates an upward bias on β_l
- **Selection Bias on** β_k: Exit before production if ω_{it} low
  - Since higher k_t will continue with lower ω_{it}'s: $\mathbb{E}\left[\omega_{it}|\omega_{t-1}, k_t, \text{no exit}\right]$ is decreasing in k_t
  - Hence ω_{it} and k_t negatively correlated *conditional* on being in sample, creating a downward bias on β_k
- 'Fixed' input: k_t, chosen last period, in firm state
- 'Variable' input: l_t, chosen this period, not in firm state

Dynamic Model

$$V(k,\omega) = \max_{\text{exit,stay}} \{V^e(k,\omega), V^c(k,\omega)\}$$
$$V^c(k,\omega) = \max_{i\geq 0} \left\{\pi(k,\omega) - c(i) + \beta\mathbb{E}\left[V(k',\omega')|\omega\right]\right\}$$
$$\text{s.t. } k' = (1-\delta)k + i$$

- k capital, i investment, ω idiosyncratic shock
- $\pi(\cdot) - c(\cdot)$ is profit *conditional* on 'variable' choices
- $V^e(k,\omega)$ is terminal value of exit
- ω first order Markov with transition probability $p(\omega'|\omega)$
- ...use t to indicate the time series data. 1 firm

Solution to Dynamic Model

Proves that under a set of assumptions including:

- $\omega_{t+1} = g(\omega_t) + \xi_t$, stochastically increasing
- ξ_t mean 0, iid
- Scalar Unobservable: ω_t

Solutions exist to the problem:

- Investment Rule: $i_t = i_t(k_t, \omega_t)$
    - $i_t(k_t, \omega_t)$ is strictly increasing in ω_t, invertible
    - Inverse investment rule: $\omega_t = h_t(k_t, i_t)$
    - Key: If labor were 'fixed', it would enter h_t
- Exit rule: $\chi_t(k_t, \omega_t)$
    - $\chi_t(k_t, \omega_t) = 1$ iff $\omega_t \geq \underline{\omega}_t(k_t)$

Olley-Pakes Estimation

- Given observables k_t, i_t, y_t, l_t, and indicator if in data set χ_t
- Estimate Cobb-Douglas production without making parametric assumptions on $\pi(\cdot), c(\cdot), h(\cdot), g(\cdot)$

$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + \omega_t + \eta_t$$
$$y_t - \beta_l l_t = \beta_0 + \beta_k k_t + \omega_t + \eta_t$$

Solution will be a 3 stage process:

i) Estimate variable coefficient β_l

ii) Control for selection

iii) Estimate fixed coefficient β_k

Step 1: $\beta_l, \hat{\phi}_t, \hat{\omega}_t$

Using the inverse investment rule:

$$y_t = \beta_0 + \beta_k k_t + \beta_l l_t + h_t(k_t, i_t) + \eta_t$$

- Use a non-parametric estimation method for $h_t(\cdot)$
- If $h_t(\cdot)$ has a constant and linear term in k_t, then h_t, β_0, β_k cannot be separately identified
  - Let $\phi_t(k_t, i_t) \equiv \beta_0 + \beta_k k_t + h_t(k_t, i_t)$
- Estimate β_l and $\hat{\phi}_t$ with a polynomial in ϕ_t:

$$y_t = \beta_l l_t + \phi_t(k_t, i_t) + \eta_t$$

- Estimate for unobservable: $\hat{\omega}_t = \hat{\phi}_t - \beta_0 - \beta_k k_t$

Step 2: $\hat{P}_t$

$$Pr(\chi_{t+1} = 1 | \omega_{t+1}) = Pr(\omega_{t+1} \geq \underline{\omega}_{t+1})$$
$$1 - F(\underline{\omega}_{t+1} | \omega_t) \equiv P_t(i_t, k_t)$$

- Non-parametric function of exit probability
- Use data on state today and exit next period
- Estimate a probit in polynomial of state variables
  - $Pr(\chi_{t+1} = 1 | k_t, i_t) = \Phi(\Lambda(k_t, i_t))$
  - For polynomial $\Lambda(\cdot)$ and cdf of normal $\Phi(\cdot)$
  - Get estimate for $\hat{P}_t$, probability of being in data at t

Conditional Expectation with Exit

Taking expectation of production function conditional on $\chi_t = 1$

$$
\begin{aligned}
\mathbb{E}\left[y_t - \beta_l l_t | \omega_{t-1}, \chi_t = 1\right] &= \mathbb{E}\left[\beta_0 + \beta_k k_t + \omega_t + \eta_t | \omega_{t-1}, \chi_t = 1\right] \\
&= \beta_0 + \beta_k k_t + \mathbb{E}\left[\omega_t | \omega_{t-1}, \chi_t = 1\right] \\
&= \beta_0 + \beta_k k_t + \mathbb{E}\left[\omega_t | \omega_{t-1}, \omega_t \geq \underline{\omega}_t\right] \\
&= \beta_0 + \beta_k k_t + \int_{\underline{\omega}_t}^{\infty} \omega_t \frac{p(\omega_t | \omega_{t-1})}{\int_{\underline{\omega}_t}^{\infty} p(\omega' | \omega_{t-1}) d\omega'} d\omega \\
&= \beta_0 + \beta_k k_t + g(\omega_{t-1}, \underline{\omega}_t(k_t))
\end{aligned}
$$

$g(\cdot)$ is a conditional expectation function

Controlling for Exit

- $\underline{\omega}_t$ is not observed
- Control for it with P_t: $\underline{\omega}_t(k_t) = f(\omega_{t-1}, P_t)$
- Use ω_{t-1} from Step 1
- Remember $\omega_{t+1} = g(\omega_t) + \xi_t$, and $\mathbb{E}[\xi_t] = \mathbb{E}[\eta_t] = 0$

$$\begin{aligned}\mathbb{E}[y_t - \beta_l l_t | \omega_{t-1}, \chi_t = 1] &= \beta_0 + \beta_k k_t + g(\omega_{t-1}, f(\omega_{t-1}, P_t)) \\ &= \beta_0 + \beta_k k_t + \tilde{g}(\phi_{t-1} - \beta_0 - \beta_k k_{t-1}, P_t)\end{aligned}$$

Step 3: β_k

From previous line, we can estimate our production function conditional on $\chi_t = 1$

$$y_t - \beta_l l_t = \beta_0 + \beta_k k_t + \tilde{g}(\phi_{t-1} - \beta_0 - \beta_k k_{t-1}, P_t) + \eta_t + \xi_t$$
$$y_t - \beta_l l_t = \beta_k k_t + \breve{g}(\phi_{t-1} - \beta_k k_{t-1}, P_t) + \eta_t + \xi_t$$

- By construction, ξ_t and η_t are uncorrelated with rhs
- Combined constants into $\breve{g}$ since β_0 not identified
  - ξ_t orthogonal to k_t since k_t chosen last period
  - ξ_t now orthogonal since we removed l_t from rhs
- Use estimates for $\hat{\phi}_{t-1}$, $\hat{P}_t$, and β_l
- Estimate β_k using a polynomial for $\breve{g}(\cdot)$ and NLLS