Sophisticated Monetary Policies

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Motivation

Ramsey Approach

- Ramsey outcome:
  - maximizes HH's time-zero utility
  - constitutes a c.e.
  - $x_t^* : \mathbb{S}^t \longrightarrow \mathbb{X}$,
    $\gamma_t^G : \mathbb{S}^t \longrightarrow \Gamma$
- Commitment to unconditional actions
- Private deviations?

Implementation via Nonexistence

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Implementation via Nonexistence

Bassetto (2005)

- Sophisticated outcome:
  - Optimality not required
  - constitutes a c.e.
  - $x_t^* : \mathbb{S}^t \times \mathbb{H}^t \longrightarrow \mathbb{X}$,
    $\gamma_t^G : \mathbb{S}^t \times \mathbb{H}^t \longrightarrow \Gamma$
- Commitment to strategy
- Private deviations?

Implementation via discouraging deviations

Requires continuation outcomes (after any history) constitute c.e.

This paper

- Uses Bassetto (2005) approach in two standard monetary models:
  - Model with One-Period Price-Setting
  - Model with Staggered Price-Setting
- Implementation with Sophisticated Policies:
  - with Reversion to a Money Regime
  - with Pure Interest-Rate Rules
  - with Reversion to a Hybrid Rule

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- Uses Bassetto (2005) approach in two standard monetary models:
  - Model with One-Period Price-Setting
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- Implementation with Sophisticated Policies:
  - with Reversion to a Money Regime
  - with Pure Interest-Rate Rules
  - with Reversion to a Hybrid Rule
- Why is this approach useful?
  - Describe policy actions after any public history
  - Off-equilibrium policy actions may guarantee *unique implementation*.

Setup of Model with One-Period Price-Setting

- Three types of agents: large number of identical consumers, a continuum of producers and a central bank
- Sources of uncertainty:
  - *flight to quality* shock $\eta_t \sim$i.i.d.$\mathcal{N}(0, \sigma_\eta^2)$
  - *velocity shock* $\upsilon_t \sim$i.i.d.$\mathcal{N}(0, \sigma_\upsilon^2)$
- Producers:
  - use labor to produce differentiated good
  - a fraction α are *flexible-price* producers and a fraction $1 - \alpha$ are *sticky-price* producers
- Central bank chooses to operate either under:
  - *money regime*: set money growth μ_t
  - *interest-rate regime*: set nominal interest rate i_t

Timing Protocol within Period

1. Sticky-price producer $j \in [0, \alpha)$ sets price $p_t^s(j)$
2. Central bank chooses monetary policy by setting one of its instruments
3. Shocks η_t and υ_t are realized
4. Flexible-price producer $j \in [\alpha, 1]$ sets price $p_t^f(j)$
5. Consumers make their decisions

Consumer behavior

(Linearized) Euler Equation:

$$y_t = E_t y_{t+1} - \psi(i_t - E_t \pi_{t+1}) + \eta_t$$

Cash-in-advance constraint:

$$\pi_t = \mu_t - (y_t - y_{t-1}) + \upsilon_t$$

Producer behavior

Optimal pricing by individual *flexible-price* producer j

$$p_t^f(j) = p_t + \gamma y_t$$

Optimal pricing by individual *sticky-price* producer j

$$p_t^s(j) = E_{t-1}\left[p_t + \gamma y_t\right]$$

Let $x_t(j) \equiv p_t^s(j) - p_{t-1}$

$$x_t(j) = E_{t-1}\left[\pi_t + \gamma y_t\right]$$

Let $x_t \equiv \frac{1}{1-\alpha}\int_\alpha^1 x_t(j)dj$

$$x_t = E_{t-1}\left[\pi_t + \gamma y_t\right]$$

Inflation Dynamics

Aggregate price level p_t is given by

$$p_t = \int_0^{\alpha} p_t^f(j)dj + \int_{\alpha}^{1} p_t^s(j)dj$$

Inflation rate evolves according to

$$\pi_t = \kappa y_t + x_t$$

where $\kappa \equiv \frac{\alpha\gamma}{1-\alpha}$

Competitive Equilibrium

Let $s^t = (s_0, ..., s_t)$, where $s_t = (\eta_t, \upsilon_t)$

Let $\delta_t(s^{t-1}) = (\delta_{1t}(s^{t-1}), \delta_{2t}(s^{t-1}))$ be a monetary policy, where

- $\delta_{1t}(s^{t-1}) \in \{M, I\}$ denotes regime choice
- $\delta_{2t}(s^{t-1})$ denotes policy choice within the regime

A **competitive equilibrium** is given by allocations $\{y_t(s^{t-1})\}$, prices $\{x_t(s^{t-1}), \pi_t(s^{t-1})\}$ and policies $\{\delta_t(s^{t-1})\}$ such that satisfy:

$$\text{(i)} \quad y_t = E_t y_{t+1} - \psi(i_t - E_t \pi_{t+1}) + \eta_t$$

$$\pi_t = \mu_t - (y_t - y_{t-1}) + \upsilon_t$$

$$\text{(ii)} \quad x_t = E_{t-1}\left[\pi_t + \gamma y_t\right]$$

$$\text{(iii)} \quad \pi_t = \kappa y_t + x_t$$

Competitive Equilibrium Cont'd

LEMMA 1: Any competitive equilibrium must satisfy

$$\pi_t(s^t) = \kappa y_t(s^t) + E_{t-1}\pi_t(s^t)$$

Also,

$$E_{t-1}y_t(s^t) = 0 \quad \text{and} \quad x_t(s^{t-1}) = E_{t-1}\pi_t(s^t)$$
$$E_{t-1}x_{t+1}(s^t) = E_{t-1}\pi_{t+1}(s^{t+1}) = E_{t-1}i_t$$

where $i_t = \begin{cases} i_t(s^{t-1}) & \text{if central bank uses an interest-rate regime at } t \\ i_t(s^t) & \text{if central bank uses a money regime at } t \end{cases}$

Sophisticated Equilibrium

Let $q_t \equiv (x_t, \delta_t, s_t, y_t, \pi_t)$.
Let $h_t \equiv (h_{t-1}, q_t)$ for $t \geq 0$ with $h_{-1} = y_{-1}$ given.
Let $h_{gt} = (h_{t-1}, x_t)$. Let $h_{yt} = (h_{t-1}, x_t, \delta_t, s_t)$

Strategies:	sticky-price producers	$\sigma_x = \{x_t(h_{t-1})\}$
	central bank:	$\sigma_g = \{\delta_t(h_{gt})\}$
Allocation rules:	output dynamics:	$\sigma_y = \{y_t(h_{yt})\}$
	inflation dynamics:	$\sigma_\pi = \{\pi_t(h_{yt})\}$

A **Sophisticated equilibrium** σ is given by (σ_x, σ_g) and (σ_y, σ_π) s.t.:

(1) given any h_{t-1}, the continuation outcome induced by σ constitutes a continuation competitive equilibrium

(2) given any h_{yt}, the continuation outcome induced by σ constitutes a continuation competitive equilibrium.

Unique Implementation

We say σ_g^* **uniquely implements** a desired competitive equilibrium $\{a_t^*(s^t)\}$ if any sophisticated equilibrium of the form $(\sigma_g^*, \sigma_x, \sigma_y, \sigma_\pi)$ coincides with the desired competitive equilibrium $\{a_t^*(s^t)\}$.

Implementation of Sophisticated Policies via Reversion to Money Regime

Sticky-price producers optimization condition:

$$x_t(h_{t-1}) = E\left[\pi_t(h_{yt}) + \gamma y_t(h_{yt}) | h_{t-1}\right]$$

$$\text{where } h_{yt} = (h_{t-1}, x_t(h_{t-1}), \delta_t(h_{t-1}, x_t(h_{t-1})), s_t)$$

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Lemma 2: For any $(h_{t-1}, \widehat{x}_t)$, if the central bank chooses money regime, then there exists a choice of money growth μ_t such that:

$$\widehat{x}_t \neq E\left[\pi_t(\widehat{h}_{yt}) + \gamma y_t(\widehat{h}_{yt})\right]$$

$$\text{where } h_{yt} = (h_{t-1}, \widehat{x}_t, M, \mu_t)$$

Unique Implementation with Money Reversion

Proposition 1: Any competitive equilibrium in which the central bank uses interest rates as its instrument can be implemented as a unique equilibrium with sophisticated policies with one-period reversion to a money regime.

Moreover, after any deviation in period t, the equilibrium outcomes from period $t+1$ are the desired outcomes.

Implementation with Pure Interest-Rate Rules

King Rule: $$i_t(s^{t-1}) = i_t^*(s^{t-1}) + \phi(x_t(s^{t-1}) - x_t^*(s^{t-1}))$$

Proposition 2: Suppose the central bank sets i_t according to the King rule. Then any of the continuum of sequences indexed by the initial condition x_0 and parameter c that satisfies:

$$\begin{aligned} x_{t+1} &= i_t + c\eta_t \\ \pi_t &= x_t + \kappa(1 + \psi c)\eta_t \\ y_t &= (1 + \psi c)\eta_t \end{aligned}$$

is a sophisticated outcome.

Implementation with a Hybrid Rule

Consider any bounded c.e. $\{x_t^*(s^{t-1}), \pi_t^*(s^{t-1}), y_t^*(s^{t-1})\}$ with an associated $i_t^*(s^{t-1})$.
Fix $\overline{x}$ and $\underline{x}$ such that $\overline{x} > \max_t x_t^*(s^{t-1})$ and $\underline{x} < \min_t x_t^*(s^{t-1})$

if $\widehat{x}_t \in [\underline{x}, \overline{x}]$, choose interest-rate regime:

$$i_t = i_t^*(s^{t-1}) + \phi(\widehat{x}_t - x_t^*(s^{t-1}))$$

otherwise, choose money regime:

$$\mu_t = \widehat{x}_t - y_{t-1}(s^{t-1}) + \frac{1+\kappa}{\kappa}\left[x_t^*(s^{t-1}) - \widehat{x}_t\right]$$

Proposition 3: Any bounded competitive equilibrium can be uniquely implemented by the King-money hybrid rule.

Moreover, after any deviation in period t, the equilibrium outcomes from period $t+1$ are the desired outcomes.