Sophisticated Monetary Policies

Andrew Atkeson, V. V. Chari and Patrick Kehoe
Quarterly Journal of Economics 2010

October 13, 2010
Motivation

Ramsey Approach

• Ramsey outcome:
  ○ maximizes HH’s time-zero utility
  ○ constitutes a c.e.
  ○ $x_t^* : S^t \rightarrow X,$
  $\gamma_t^G : S^t \rightarrow \Gamma$

• Commitment to unconditional actions

• Private deviations?

Implementation via Nonexistence

Andrew Atkeson, V. V. Chari and Patrick Kehoe Quarterly Journal of Economics 2010 ()
Sophisticated Monetary Policies

October 13, 2010 2 / 16
Motivation

Ramsey Approach

- Ramsey outcome:
  - maximizes HH’s time-zero utility
  - constitutes a c.e.
    \[ x^*_t : S^t \rightarrow X, \]
    \[ \gamma^G_t : S^t \rightarrow \Gamma \]
- Commitment to unconditional actions
- Private deviations?

Bassetto (2005)

- Sophisticated outcome:
  - Optimality not required
  - constitutes a c.e.
    \[ x^*_t : S^t \times H^t \rightarrow X, \]
    \[ \gamma^G_t : S^t \times H^t \rightarrow \Gamma \]
- Commitment to strategy
- Private deviations?

Implementation via Nonexistence

Implementation via discouraging deviations

Requires continuation outcomes (after any history) constitute c.e.
This paper

- Uses Bassetto (2005) approach in two standard monetary models:
  - Model with One-Period Price-Setting
  - Model with Staggered Price-Setting

- Implementation with Sophisticated Policies:
  - with Reversion to a Money Regime
  - with Pure Interest-Rate Rules
  - with Reversion to a Hybrid Rule
This paper

- Uses Bassetto (2005) approach in two standard monetary models:
  - Model with One-Period Price-Setting
  - Model with Staggered Price-Setting

- Implementation with Sophisticated Policies:
  - with Reversion to a Money Regime
  - with Pure Interest-Rate Rules
  - with Reversion to a Hybrid Rule

- Why is this approach useful?
  - Describe policy actions after any public history
  - Off-equilibrium policy actions may guarantee *unique implementation*.
Setup of Model with One-Period Price-Setting

- Three types of agents: large number of identical consumers, a continuum of producers and a central bank

- Sources of uncertainty:
  - *flight to quality* shock $\eta_t \sim \text{i.i.d.} \mathcal{N}(0, \sigma_\eta^2)$
  - *velocity shock* $\nu_t \sim \text{i.i.d.} \mathcal{N}(0, \sigma_\nu^2)$

- Producers:
  - use labor to produce differentiated good
  - a fraction $\alpha$ are *flexible-price* producers and a fraction $1 - \alpha$ are *sticky-price* producers

- Central bank chooses to operate either under:
  - *money regime*: set money growth $\mu_t$
  - *interest-rate regime*: set nominal interest rate $i_t$
1. Sticky-price producer $j \in [0, \alpha)$ sets price $p^s_t(j)$

2. Central bank chooses monetary policy by setting one of its instruments

3. Shocks $\eta_t$ and $\nu_t$ are realized

4. Flexible-price producer $j \in [\alpha, 1]$ sets price $p^f_t(j)$

5. Consumers make their decisions
Consumer behavior

(Linearized) Euler Equation:

\[ y_t = E_t y_{t+1} - \psi (i_t - E_t \pi_{t+1}) + \eta_t \]

Cash-in-advance constraint:

\[ \pi_t = \mu_t - (y_t - y_{t-1}) + \upsilon_t \]
Producer behavior

Optimal pricing by individual *flexible-price* producer $j$

\[ p^f_t(j) = p_t + \gamma y_t \]

Optimal pricing by individual *sticky-price* producer $j$

\[ p^s_t(j) = E_{t-1} [p_t + \gamma y_t] \]

Let $x_t(j) \equiv p^s_t(j) - p_{t-1}$

\[ x_t(j) = E_{t-1} [\pi_t + \gamma y_t] \]

Let $x_t \equiv \frac{1}{1-\alpha} \int_{\alpha}^{1} x_t(j) dj$

\[ x_t = E_{t-1} [\pi_t + \gamma y_t] \]
Inflation Dynamics

Aggregate price level $p_t$ is given by

$$p_t = \int_0^\alpha p^f_t(j)\,dj + \int_\alpha^1 p^s_t(j)\,dj$$

Inflation rate evolves according to

$$\pi_t = \kappa y_t + \chi_t$$

where $\kappa \equiv \frac{\alpha \gamma}{1 - \alpha}$
Competitive Equilibrium

Let \( s^t = (s_0, ..., s_t) \), where \( s_t = (\eta_t, \nu_t) \)

Let \( \delta_t(s^{t-1}) = (\delta_1(s^{t-1}), \delta_2(s^{t-1})) \) be a monetary policy, where

- \( \delta_1(s^{t-1}) \in \{M, I\} \) denotes regime choice
- \( \delta_2(s^{t-1}) \) denotes policy choice within the regime

A competitive equilibrium is given by allocations \( \{y_t(s^{t-1})\} \), prices \( \{x_t(s^{t-1}), \pi_t(s^{t-1})\} \) and policies \( \{\delta_t(s^{t-1})\} \) such that satisfy:

(i) \[ y_t = E_t y_{t+1} - \psi(i_t - E_t \pi_{t+1}) + \eta_t \]
    \[ \pi_t = \mu_t - (y_t - y_{t-1}) + \nu_t \]

(ii) \[ x_t = E_{t-1} [\pi_t + \gamma y_t] \]

(iii) \[ \pi_t = \kappa y_t + x_t \]
LEMMA 1: Any competitive equilibrium must satisfy

\[ \pi_t(s^t) = \kappa y_t(s^t) + E_{t-1}\pi_t(s^t) \]

Also,

\[ E_{t-1}y_t(s^t) = 0 \quad \text{and} \quad x_t(s^{t-1}) = E_{t-1}\pi_t(s^t) \]
\[ E_{t-1}x_{t+1}(s^t) = E_{t-1}\pi_{t+1}(s^{t+1}) = E_{t-1}i_t \]

where \( i_t = \begin{cases} 
  i_t(s^{t-1}) & \text{if central bank uses an interest-rate regime at } t \\
  i_t(s^t) & \text{if central bank uses a money regime at } t 
\end{cases} \)
Sophisticated Equilibrium

Let \( q_t \equiv (x_t, \delta_t, s_t, y_t, \pi_t) \).

Let \( h_t \equiv (h_{t-1}, q_t) \) for \( t \geq 0 \) with \( h_{-1} = y_{-1} \) given.

Let \( h_{gt} = (h_{t-1}, x_t) \). Let \( h_{yt} = (h_{t-1}, x_t, \delta_t, s_t) \)

Strategies:

sticky-price producers
\[ \sigma_x = \{x_t(h_{t-1})\} \]

central bank:
\[ \sigma_g = \{\delta_t(h_{gt})\} \]

Allocation rules:

output dynamics:
\[ \sigma_y = \{y_t(h_{yt})\} \]

inflation dynamics:
\[ \sigma_\pi = \{\pi_t(h_{yt})\} \]

A **Sophisticated equilibrium** \( \sigma \) is given by \((\sigma_x, \sigma_g)\) and \((\sigma_y, \sigma_\pi)\) s.t.:

(1) given any \( h_{t-1} \), the continuation outcome induced by \( \sigma \) constitutes a
continuation competitive equilibrium

(2) given any \( h_{yt} \), the continuation outcome induced by \( \sigma \) constitutes a
continuation competitive equilibrium.
We say $\sigma^*_g$ uniquely implements a desired competitive equilibrium $\{a^*_t(s^t)\}$ if any sophisticated equilibrium of the form $(\sigma^*_g, \sigma_x, \sigma_y, \sigma_\pi)$ coincides with the desired competitive equilibrium $\{a^*_t(s^t)\}$. 
Implementation of Sophisticated Policies via Reversion to Money Regime

Sticky-price producers optimization condition:

\[ x_t(h_{t-1}) = E \left[ \pi_t(h_{yt}) + \gamma y_t(h_{yt}) \mid h_{t-1} \right] \]

where \( h_{yt} = (h_{t-1}, x_t(h_{t-1}), \delta_t(h_{t-1}, x_t(h_{t-1})), s_t) \)
Implementation of Sophisticated Policies via Reversion to Money Regime

Sticky-price producers optimization condition:

\[ x_t(h_{t-1}) = E \left[ \pi_t(h_{yt}) + \gamma y_t(h_{yt}) \middle| h_{t-1} \right] \]

where \( h_{yt} = (h_{t-1}, x_t(h_{t-1}), \delta_t(h_{t-1}, x_t(h_{t-1})), s_t) \)

**Lemma 2**: For any \((h_{t-1}, \hat{x}_t)\), if the central bank chooses money regime, then there exists a choice of money growth \( \mu_t \) such that:

\[ \hat{x}_t \neq E \left[ \pi_t(\hat{h}_{yt}) + \gamma y_t(\hat{h}_{yt}) \right] \]

where \( h_{yt} = (h_{t-1}, \hat{x}_t, M, \mu_t) \)
**Proposition 1:** Any competitive equilibrium in which the central bank uses interest rates as its instrument can be implemented as a unique equilibrium with sophisticated policies with one-period reversion to a money regime.

Moreover, after any deviation in period $t$, the equilibrium outcomes from period $t + 1$ are the desired outcomes.
Implementation with Pure Interest-Rate Rules

**King Rule:**
\[
i_t(s^{t-1}) = i_t^*(s^{t-1}) + \phi(x_t(s^{t-1}) - x_t^*(s^{t-1}))
\]

**Proposition 2:** Suppose the central bank sets \(i_t\) according to the King rule. Then any of the continuum of sequences indexed by the initial condition \(x_0\) and parameter \(c\) that satisfies:

\[
\begin{align*}
  x_{t+1} &= i_t + c\eta_t \\
  \pi_t &= x_t + \kappa(1 + \psi c)\eta_t \\
  y_t &= (1 + \psi c)\eta_t
\end{align*}
\]

is a sophisticated outcome.
Implementation with a Hybrid Rule

Consider any bounded c.e. \( \{ x_t^*(s^{t-1}), \pi_t^*(s^{t-1}), y_t^*(s^{t-1}) \} \) with an associated \( i_t^*(s^{t-1}) \).

Fix \( \bar{x} \) and \( \underline{x} \) such that \( \bar{x} > \max_t x_t^*(s^{t-1}) \) and \( \underline{x} < \min_t x_t^*(s^{t-1}) \).

if \( \hat{x}_t \in [\underline{x}, \bar{x}] \), choose interest-rate regime:

\[
i_t = i_t^*(s^{t-1}) + \phi(\hat{x}_t - x_t^*(s^{t-1}))
\]

otherwise, choose money regime:

\[
\mu_t = \hat{x}_t - y_{t-1}(s^{t-1}) + \frac{1+\kappa}{\kappa} \left[ x_t^*(s^{t-1}) - \hat{x}_t \right]
\]

**Proposition 3:** Any bounded competitive equilibrium can be uniquely implemented by the King-money hybrid rule.

Moreover, after any deviation in period \( t \), the equilibrium outcomes from period \( t + 1 \) are the desired outcomes.