“Inferring Labor Income Risk From Economic Choices: An Indirect Inference Approach”

by Fatih Guvenen and Anthony Smith

October 27, 2010
Motivation

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One is that the rise in inequality stems from the accumulation of persistent idiosyncratic shocks.

The alternative is a systematic fanning out of earnings over time, which can represent uncertainty if individuals have to learn their growth rate.
Overview

- This paper will assume that individuals face the following income process

\[ y_t^i = g(t, \text{observables}, \ldots) + \alpha^i + \beta^i t + \underbrace{z_t^i + \epsilon_t^i \quad \text{in (1)}}_{\text{stochastic component}} \]

where \( z_t^i = \rho z_{t-1}^i + \eta_t^i \); \( \eta_t^i \) and \( \epsilon_t^i \) are mean zero innovations.
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- The auxiliary model will be motivated by the model above under quadratic utility and no borrowing constraints.
The Learning Problem

- The assume that $\beta^i = \beta_k^i + \beta_u^i$ where $\beta_k^i$ is observed and $\sigma_\beta^2 = \sigma_{\beta_k}^2 + \sigma_{\beta_u}^2$. Let $\lambda \equiv \frac{\sigma_{\beta_u}^2}{\sigma_\beta^2}$. 
The Learning Problem

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• Learning $\beta^i$ can then be written as a Kalman Filter problem with state equation

$$
\begin{bmatrix}
\beta^i \\
z^i_{t+1}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
0 & \rho
\end{bmatrix}
\begin{bmatrix}
\beta^i \\
z^i_t
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\eta^i_{t+1}
\end{bmatrix}
$$

With a second observation equation

$$
\tilde{y}^i_t \equiv y^i_t - \alpha^i - \epsilon^i_t = [t \quad 1]
\begin{bmatrix}
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\end{bmatrix}
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• For future reference we will define $\hat{x}_t^i = \tilde{y}_t^i - \mathbb{E}_{t-1}(\tilde{y}_t^i)$
The Linear-Quadratic Framework

- The consumption-savings problem in the quadratic utility framework can be written as

\[ V_i(t) = \max_{C_t, a_{t+1}} \left\{ -(C_t - C^*)^2 + \frac{1}{1 + r} \mathbb{E}_t \left[ V_{t+1}(a_{t+1}, \hat{\beta}_{t+1}, \hat{z}_{t+1}) \right] \right\} \]

subject to

\[ C_t + a_{t+1} = (1 + r)a_t + Y_t \]

and the Kalman Recursions, where \( Y_t = \alpha_t + \beta_t t + z_t \).
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- Note that this can incorporate both RIP and HIP processes.
- One can see that the optimal consumption satisfies

$$\Delta C_t = \varphi_t \left[ \sum_{s=0}^{T-t} \left( \frac{1}{1+r} \right)^s (\mathbb{E}_t - \mathbb{E}_{t-1}) Y_{t+s} \right]$$
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\]

- All new information comes from \( \hat{\xi}_t^i \), thus one can obtain

\[
\Delta C_t = \Pi_t \times \hat{\xi}_t^i
\]

and derive the level of optimal consumption

\[
C_t^i = \varphi_t w_t^i + \frac{1}{1 + r} \Phi_{t+1} \hat{\beta}_t^i + \frac{1}{1 + r} \rho \Psi_{t+1} \hat{z}_t^i
\]
Some Examples

From this framework one can see that individuals with different beliefs about their growth rate $\beta$ will respond differently to the save earnings innovation. This will help determine beliefs in the estimation.
**Proposition**

_Controlling for and individuals income and assets, the consumption level is an increasing function of the individuals beliefs \( \hat{\beta}_t^i \)._
Some Examples

Consumption decisions can also be informative about the degree of private information, $\lambda$. 

If $\lambda=1$: $C_{t=3}^1 = C_{t=3}^2$

If $\lambda<1$: $C_{t=3}^1 > C_{t=3}^2$

Forecast paths under HIP with $\lambda<1$

Forecast path for **both** agents under HIP with $\lambda=1$
Full Consumption Savings Model

- Agents live $T$ periods and retire at date $R < T$. The Dynamic Program for the full consumption savings model is

$$V_t^i(w_t^i, \hat{\beta}_t^i, \hat{\alpha}_t^i) = \max_{C_t^i, a_{t+1}^i} \left\{ \frac{(C_t^i)^{1-\phi}}{1-\phi} + \delta E_t \left[ V_{t+1}^i(w_{t+1}^i, \hat{\beta}_{t+1}^i, \hat{\alpha}_{t+1}^i) \right] \right\}$$

subject to

$$C_t^i + a_{t+1}^i = w_t^i$$
$$w_t^i = (1 + r)a_t^i + Y_t^i$$
$$Y_t^i = \overline{Y} + \exp(\overline{y}_t + \alpha_t^i + \beta_t^i + \alpha_t^i)$$
$$a_{t+1}^i \geq a_t^i,$$ and Kalman Recursions

$\overline{y}_t$ is the population average of log income at age $t$.

- For retirement we have, where $Y_t = \Phi(Y_R^i, \overline{Y})$

$$V_t^i(w_t^i, Y_R^i) = \max_{C_t^i, a_{t+1}^i} \left\{ \frac{(C_t^i)^{1-\phi}}{1-\phi} + \delta V_{t+1}^i(w_{t+1}^i; Y_R^i) \right\}$$
The Data

• They use Blundell, Pistaferri and Preston method for imputing consumption in the PSID using food demand estimates from the CEX.
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- They also extend the method to cover the period from 1968 to 1992 (instead of 1980 - 1992).
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- They use Blundell, Pistaferri and Preston method for imputing consumption in the PSID using food demand estimates from the CEX.
- The basic approach involves estimating a demand system for food consumption as a function of non-durable expenditures, demographic variables, and relative prices.
- They also extend the method to cover the period from 1968 to 1992 (instead of 1980 - 1992).
- They convert the Data to Per-Adult Equivalent Units by regressing each variable on family size, a race dummy, a region dummy, a dummy indicating whether the head is employed, a dummy indicating residence in a large city, and a set of cohort dummies. Use the residuals in the analysis below.
The Auxiliary Model

• Ideally they would want to use the equations from the linear-quadratic framework for the auxiliary model, but these equations depend on $\hat{\beta}_{t|t-1}$ and $2_{t|t-1}$. 

They also divide the population into 2 groups: between 25 and 38, and between 39 and 55. Allow the coefficients to vary across these two groups.
The Auxiliary Model

- Ideally they would want to use the equations from the linear-quadratic framework for the auxiliary model, but these equations depend on $\hat{\beta}_t^{i|t-1}$ and $\hat{\gamma}_t^{i|t-1}$.
- Thus they use the following equations for the auxiliary model:

$$
c_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t+1} + a_4 y_{t+2} + a_5 \bar{y}_{1,t-3} + a_6 \bar{y}_{t+3,R} \\
+ a_7 \Delta y_{1,t-3} + a_8 \Delta y_{t+3,R} + a_9 c_{t-1} + a_{10} c_{t-2} + a_{11} c_{t+1} + a_{12} c_{t+2} + \epsilon^c_t
$$

and

$$
y_t = b_0 + b_1 y_{t-1} + b_2 y_{t-2} + b_3 y_{t+1} + b_4 y_{t+2} + b_5 \bar{y}_{1,t-3} + b_6 \bar{y}_{t+3,R} \\
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$$y_{t,*}^i = y_t^i + u_{t,y}^i$$
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where $u_{t,y}^i$ and $u_{t,c}^i$ are mean 0, and $\bar{u}_{t,c}^i$ is not in order to capture level differences from the per-adult equivalent estimation and initial wealth levels.
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- Fill in missing data using a reasonable procedure to match percentiles. Will still be consistent if same procedure is applied to real and simulated data.
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  where $u_{t,y}^i$ and $u_{t,c}^i$ are mean 0, and $\bar{u}_{i,c}^i$ is not in order to capture level differences from the per-adult equivalent estimation and initial wealth levels.
- Fill in missing data using a reasonable procedure to match percentiles. Will still be consistent if same procedure is applied to real and simulated data.
- They compute the Wealth to income ratio in 1984 and 1989, average and add the additional moment

  $10 \times (W Y_{PSID} - W Y_{Model})^2$

  to the objective function.
Estimation

- Used the following objective function for the data

\[ \mathcal{L} = |\Sigma|^{-J/2} \exp \left( -\frac{1}{2} \sum_i \sum_t \epsilon_{t,Data} \Sigma^{-1} (\epsilon_{t,Data})' \right) \]
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- They let \( \hat{a}, \hat{b} \) and \( \hat{\Sigma} \) be the reduced form parameters that maximize \( \mathcal{L} \), then for a given set of structural parameters, \( \theta \), they simulate the model to match exactly the number of observations and missing data pattern \( N \) times
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• For each simulation they obtain \( \tilde{a}_n, \tilde{b}_n \) and \( \tilde{\Sigma}_n \), which they average to get \( \tilde{a}(\theta) \), etc.
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- The estimated structural parameters \( \hat{\theta} \) are then chosen by

\[
\hat{\theta} = \arg\min_{\theta} \left[ L(\hat{a}, \hat{b}, \hat{\Sigma}, PSID) - L(\tilde{a}(\theta), \tilde{b}(\theta), \tilde{\Sigma}(\theta), PSID) \right]
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• Use Monte-Carlo to study small sample bias and identification. Find strong evidence for local identification.
Estimation results

- Note that lambda is estimated to be around 0.345 which means that \( \frac{\sigma^2_{\beta_k}}{\sigma^2_\beta} = 1 - \lambda^2 = .880 \).
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- Also estimated discount factor to some precision. This, however, is condition on the fixed value of \( \phi. \) \( \delta \) is quite sensitive to the preset value of risk aversion.

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>( \sigma_\alpha )</td>
<td>0.284 (0.027)</td>
</tr>
<tr>
<td>( \sigma_\beta )</td>
<td>1.852 (0.188)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-0.162 (0.151)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.754 (0.025)</td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>0.196 (0.005)</td>
</tr>
<tr>
<td>( \sigma_\epsilon )</td>
<td>0.004 (0.021)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.345 (0.074)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.950 (0.001)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.874 (0.083)</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>0.147 (0.007)</td>
</tr>
<tr>
<td>( \sigma_e )</td>
<td>0.356 (0.002)</td>
</tr>
<tr>
<td>( \sigma_{e_0} )</td>
<td>0.428 (0.009)</td>
</tr>
<tr>
<td>max % constrained..</td>
<td>12.7%</td>
</tr>
<tr>
<td>.. at age</td>
<td>30</td>
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<td>( \theta_{50} ) / mean income</td>
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Also estimated discount factor to some precision. This, however, is condition on the fixed value of $\phi$. $\delta$ is quite sensitive to the preset value of risk aversion.

They also did some analysis on the goodness of fit of the model. Over 100 repetitions of the objective function the test statistic has a mean of 2.29 and a maximum value of 4.03. The PSID data yielded a value of 26.37.
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- They also did some analysis on the goodness of fit of the model. Over 100 repetitions of the objective function the test statistic has a mean of 2.29 and a maximum value of 4.03. The PSID data yielded a value of 26.37.
- This is because the model falls short in matching the auxiliary consumption model.

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<td>0.356</td>
</tr>
<tr>
<td>( \sigma_{c0} )</td>
<td>0.428</td>
</tr>
<tr>
<td>( \text{max % constrained} )</td>
<td>12.7%</td>
</tr>
<tr>
<td>( \text{at age} )</td>
<td>30</td>
</tr>
<tr>
<td>( \bar{a}_0 ) / mean income</td>
<td>0.33</td>
</tr>
<tr>
<td>( \bar{a}_{55} ) / mean income</td>
<td>0.53</td>
</tr>
</tbody>
</table>
# Goodness of Fit

## Panel A: Income Equation

<table>
<thead>
<tr>
<th></th>
<th>constant</th>
<th>(y_{t-1})</th>
<th>(y_{t-2})</th>
<th>(y_{t+1})</th>
<th>(y_{t+2})</th>
<th>(\bar{y}_{1,t-3})</th>
<th>(\bar{y}_{t+3,T})</th>
<th>(\Delta y_{1,t-3})</th>
<th>(\Delta y_{t+3,T})</th>
<th>(c_{t-1})</th>
<th>(c_{t-2})</th>
<th>(c_{t+1})</th>
<th>(c_{t+2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Group:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(1) Data</td>
<td>-0.036**</td>
<td>0.346</td>
<td>0.360</td>
<td>0.077</td>
<td>0.097</td>
<td>-0.098</td>
<td>0.150</td>
<td>0.037</td>
<td>-0.022</td>
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<td></td>
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</tr>
<tr>
<td>(2) Model</td>
<td>-0.026tti</td>
<td>0.337</td>
<td>0.357</td>
<td>0.093</td>
<td>0.102</td>
<td>-0.081</td>
<td>0.124</td>
<td>-0.016tti</td>
<td>-0.021</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Middle-Age Group:</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>(3) Data</td>
<td>0.006**</td>
<td>0.418</td>
<td>0.358</td>
<td>0.111</td>
<td>0.093</td>
<td>-0.027</td>
<td>0.043**</td>
<td>0.031</td>
<td>0.028</td>
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<tr>
<td>(4) Model</td>
<td>-0.001tti</td>
<td>0.401</td>
<td>0.375</td>
<td>0.122</td>
<td>0.105</td>
<td>-0.038</td>
<td>0.022</td>
<td>0.012</td>
<td>0.045</td>
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</table>

## Panel B: Consumption Equation

<table>
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<tr>
<th></th>
<th>constant</th>
<th>(y_{t-1})</th>
<th>(y_{t-2})</th>
<th>(y_{t+1})</th>
<th>(y_{t+2})</th>
<th>(\bar{y}_{1,t-3})</th>
<th>(\bar{y}_{t+3,T})</th>
<th>(\Delta y_{1,t-3})</th>
<th>(\Delta y_{t+3,T})</th>
<th>(c_{t-1})</th>
<th>(c_{t-2})</th>
<th>(c_{t+1})</th>
<th>(c_{t+2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Group:</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>(5) Data</td>
<td>-0.007</td>
<td>0.108</td>
<td>0.042*</td>
<td>-0.023</td>
<td>-0.005</td>
<td>-0.045*</td>
<td>-0.017</td>
<td>0.030</td>
<td>-0.002</td>
<td>0.248</td>
<td>0.262</td>
<td>0.178</td>
<td>0.175</td>
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<tr>
<td>(6) Model</td>
<td>-0.021tti</td>
<td>0.105</td>
<td>0.079†</td>
<td>-0.022</td>
<td>-0.039</td>
<td>-0.079</td>
<td>0.025</td>
<td>0.001</td>
<td>0.013</td>
<td>0.208tti</td>
<td>0.205tti</td>
<td>0.248tti</td>
<td>0.229tti</td>
</tr>
<tr>
<td>Middle-Age Group:</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>(7) Data</td>
<td>-0.004</td>
<td>0.136</td>
<td>0.046**</td>
<td>-0.014</td>
<td>-0.040*</td>
<td>-0.082**</td>
<td>0.012</td>
<td>0.030</td>
<td>0.028</td>
<td>0.270</td>
<td>0.260</td>
<td>0.177</td>
<td>0.187</td>
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<td>(8) Model</td>
<td>0.007tti</td>
<td>0.103</td>
<td>0.041</td>
<td>0.009</td>
<td>-0.044</td>
<td>0.050tti</td>
<td>-0.042tti</td>
<td>0.027</td>
<td>0.041</td>
<td>0.197tti</td>
<td>0.211tti</td>
<td>0.224tti</td>
<td>0.257tti</td>
</tr>
</tbody>
</table>
Implications

Income Inequality

- US Data, **Cleaned** of Cohort Effects
- Benchmark Estimated Model
- US Data, **Uncleaned** (for Comparison)
- Forecast Variance, Benchmark Model
Risk Aversion and Time Preference

- Cannot separately identify $\delta$ and $\phi$, using Monte Carlo and 200 observations find a correlation of the parameters to be -0.88. Suggests only a combination is identified.
- They do some sensitivity analysis and find that one can either fix $\delta$ or $\phi$, but fixing both will create severe biases. Finally, they find that the main role of the Wealth to Income moment is to pin down $\delta$. 