Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations

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November 3, 2010

Outline

- Identify the difficulty of using random search models an environment with aggregate dynamics.
- Build a model of directed search with aggregate shocks to productivity.
- Define Block Recursive Equilibrium and sketch existence proof.
- Extend model to allow for ex-ante heterogeneous workers.

Punchline: Directed search models are better suited to study search in an environment with aggregate dynamics.

Random Search

Models of random search have been used to study

- unemployment duration
- shape and extent of wage dispersion
- job transitions and occupational mobility
- change in earnings over the life cycle.

But what happens outside of steady state?

Random search on the job with wage posting

- Wage posting: a job offer is associated with a fixed wage.
- Job offers arrive at a Poisson rate to unemployed and employed workers (typically at different rates).
- Unemployed workers accept any wage that is greater than the value of continuing search.
- To compute acceptance rule, unemployed worker needs to know the wage distribution.
- Employed workers accept any wage that is great than their current wage.

Random search, cont.

Firms make zero profits. Firms choose wage to balance two opposing considerations:

- Conditional on a job being filled, a higher wage translates into lower profits.
- A higher wage increases the probability the job is filled, and conditional on the job being filled, increases the expected duration of the match before the worker is "poached" by another firm.

Firm needs to know the distribution of workers across states and the wage distribution to calculate expected profits.

Random search and aggregate dynamics

Workers and firms solve a more complicated in a non-stationary environment.

- Unemployed workers need to forecast the dynamics of the wage distribution to compute an acceptance rule.
- Firms need to forecast the dynamics of the entire wage distribution and the distribution of workers across states to compute acceptance probabilities and the expected duration of the match.

(Solution: get a bigger computer?)

Block Recursive Equilibrium

Authors develop a model of *directed* search on the job with aggregate productivity shocks.

- Agents only need to forecast the evolution of aggregate productivity shocks.
- Solve a system of functional equations in which unknown equations (value and policy functions) depend only on a one-dimensional argument.

This is what the authors refer to as a Block Recursive Equilibrium.

Workers

- Measure unity of workers, positive measure of firms.
- Workers endowed with single indivisible unit of labor.
- Workers maximize sum of discounted period utilities, $\sum_{t=0}^{\infty} \beta^t \upsilon(c_t)$
- Discount factor β.
- υ satisfies $\upsilon' \in [\underline{\upsilon}', \overline{\upsilon}']$, $\underline{\upsilon}' > 0$, $\upsilon'' \leq 0$.

Firms and technology

- Firms operate a CRS production function.
- Turns a single unit of labor into $y + z$ units of output.
- y is common to all firms, lies in set Y
- z is specific to firm-worker pair, lies in set Z.
- Firms maximize the sum of discounted period returns
- Discount factor β.

Search environment

- Labor markets are organized into a continuum of submarkets indexed by $x \in [\underline{x}, \overline{x}] = X$.
- x is the lifetime utility offered by firms to workers.
- In submarket x, ratio of vacancies to job searchers is given by tightness $\theta(x) > 0$, determined in equilibrium.

State

State of the economy is given by $\psi = (y, u, g)$.

- y is the common component of aggregate productivity.
- u is the measure of unemployed workers, $u \in [0, 1]$
- $g : X \times Z \to [0, 1]$.

$g(V, z)$ denotes the measure of workers who are employed at jobs that given them lifetime utility $\tilde{V} \leq V$ with idiosyncratic productivity component $\tilde{z} \leq z$.

Timing

Each period has four stages: separation, search, matching, production.

1. Separation

- Match is dissolved with probability $\delta \in (0, 1]$.
- Employed worker can voluntarily move into unemployment.

2. Search

- Workers (employed and unemployed) choose a submarket to apply for a job.
- Firms decide the number of vacancies to create and in which markets to place them.
- Vacancy cost $k > 0$, $\theta(x)$ taken as given.

Timing, cont.

3. Matching

- A worker meets a vacancy with probability $p(\theta(x))$
- p strictly increasing and concave, $p(0) = 0$, $p'(0) > 0$.
- A vacancy meets a worker with probability $q(\theta(x))$.
- q strictly decreasing and convex, $q(\theta) = p(\theta)/\theta$, $q(0) = 1$, $q'(0) < 0$, $p(q^{-1}(\theta))$ concave.
- If worker rejects a job offer off the equilibrium path, returns to previous position.
- If worker accepts, the match forms with match-specific productivity z_0.

Timing, cont.

4. Production

- Unemployed worker produces and eats $b > 0$ units of output.
- A worker employed at job z produces $y + z$ units of output, consumes w, where w is specified by the labor contract.

At the end of the period, Nature draws $\hat{y}$ from $\Phi_y(\hat{y}|y)$ and $\hat{z}$ from $\Phi_z(\hat{z}|z)$. Draws from $\Phi_z(\hat{z}|z)$ are independent across matches.

Contractual environment

Two environments are considered.

1. Dynamic Contracts

Firm commits to a wage that is contingent on the entire history of

- idiosyncratic component of the match
- aggregate state of the economy
- realizations of a lottery that is drawn at the beginning of each production stage.

Write recursively. Lifetime utility is an auxiliary value function.

Generalization of Burdett and Coles (2003) and Shi (2009) to environment with stochastic productivity.

Contractual environment

2. Fixed-Wage Contracts

- Firm commits to a wage that remains constant for the entire duration of the match.
- Wage is the outcome of a lottery that takes place at the formation of the match.

Analogous to Burdett and Mortensen (1998).

We will henceforth restrict our attention to BRE with dynamic contracts.

Block Recursive Equilibrium: Objects

- $U(y)$ is the lifetime utility of an unemployed worker with aggregate productivity component y.
- $J(V, y, z)$ is the profits of a firm that employer a worker, where V is the lifetime value to the worker specified by the contract, y is the aggregate productivity component, and z is the idiosyncratic match productivity draw.
- $R(V, y)$ is the lifetime utility of a worker at the beginning of the search stage, given that the worker's current contract promises lifetime utility V and the aggregate productivity component is y.
- $\theta(x, y)$ is the equilibrium market tightness in submarket x.
- $\Phi_\psi(\hat{\psi}|\psi)$ is the transition probability for the aggregate state of the economy ψ.

Definition

A Block Recursive Equilibrium is a list of value and policy functions (θ, R, m, U, J, c) together with a transition probability function Φ_ψ that satisfy:

1. For all $(V, \psi) \in X \times \Psi$,

$$R(V, y) = \max_x p(\theta(x, y))x + (1 - p(\theta(x, y)))V$$

   and m is the associated policy function.

2. For all $\psi \in \Psi$,

$$U(y) = \upsilon(b) + \beta \mathbb{E} R(U(\hat{y}), \hat{y}).$$

Definition, continued

3. For all $(V, \psi, z) \in X \times \Psi \times Z$,

$$J(V, y, z) = \max_{w, \hat{V}} y + z - w +$$
$$\beta \mathbb{E}\left[(1 - d(\hat{y}, \hat{z}))(1 - \tilde{p}(\hat{y}, \hat{z}))J(\hat{V}(\hat{y}, \hat{z}), \hat{y}, \hat{z})\right],$$

subject to the constraints

$$V = \upsilon(w) + \beta \mathbb{E}\left[d(\hat{y}, \hat{z})R(U(\hat{y}), \hat{y}) + (1 - d(\hat{y}, \hat{z}))\hat{V}(\hat{y}, \hat{z})\right],$$

$$d(\hat{y}, \hat{z}) \begin{cases} 1 & \text{if } U(y) > V(y, z) \\ \delta & \text{if else,} \end{cases}$$

where $\tilde{p}(\hat{y}, \hat{z}) = p(\theta(m(\hat{V}(\hat{y}, \hat{z}))))$ and c is the associated policy function.

Definition, continued

4. For all $(x, \psi) \in X \times \Psi$,

$$k \geq q(\theta(x, y))J(x, y)$$

and $\theta(x, y) \geq 0$, with complementary slackness.

5. For all $\psi \in \Psi$, Φ_Ψ is consistent with transition probability of the exogenous variables, y and z, and with the policy functions m and c.

Remarks

- Note that the agents' value and policy functions (θ, R, m, U, J, c) depend on the aggregate state of the economy only through y, and not on the distribution of workers across states, (u, g).
- To solve for the BRE, need to solve a system of functional equations in which the argument of the functions depend on three real numbers (individual state variables, V, and z).
- The block of equilibrium conditions that describe the agents' value and policy functions can be solved before having solved the law of motion of the aggregate state.

BRE: Existence

Theorem 2

A Block Recursive Equilibrium exists.

- Proved in Menzio and Shi (2010b)
- BRE does not exist for models of random search.

Intuition

Rewrite the vacancy creation condition for the firm as:

$$k \geq \max_{x \in X} q(\theta(\psi))\mathbb{I}(x, \psi) J(x, \psi, z_0), \tag{1}$$

where $\theta(\psi) \geq 0$ and $\mathbb{I}(x, \psi)$ is the probability that a randomly selected worker is willing to accept a match offering lifetime utility x.

For a randomly selected worker, $\mathbb{I}(x, \psi)$ depends on the entire distribution of workers across states. So the distribution of workers across states enters the firm's maximization problem.

Intuition, cont.

Consider a a worker who directs his search towards vacancy offering lifetime utility x.

- The probability, $\mathbb{I}(x, \psi)$, that a worker accepts a vacancy from submarket x is equal to one.
- Value to firm of filling a vacancy only depends on number of applicants, not distribution of workers across states.
- Constant vacancy cost implies that $\theta(x)$ is independent of distribution of workers.
- Probability that a worker is matched in submarket x is determined by $\theta(x)$.
- Probability that a worker matched in submarket x moves to a job in some other submarket x' is determined by $\theta(x')$.

Hence, value functions and decision rules do not depend on the distribution of workers across states.

Ex-ante Heterogeneous workers

- Continuum of workers of type s_i with measure π_i, $\sum \pi_i = 1$, $i = 1, \ldots, N$, and $s_1 < s_2 < \ldots < s_N$.
- Worker of type i produces $y + s_i + z$ units of output.
- Submarkets are indexed by the vector $\mathbf{x}$, where the ith component x_i is the lifetime utility offered to a worker of type i.
- Denote $\theta(\mathbf{x})$ to be the vacancy-to-applicant ratio in submarket $\mathbf{x}$.
- Denote $\phi(\mathbf{x})$ to be the distribution of applicants across different types.
- Let $U(i, y)$, $R(i, V, y)$, and $m(i, V, y)$ denote the value and policy functions for a worker of type i.
- Let $J(i, V, z, y)$ denote the profit function of a firm employing a worker of type i.

Theorem 3

Let $(\theta_i, R_i, m_i, U_i, J_i, c_i)$ be a BRE for the economy with ex-ante homogeneous workers of type i. There exists a BRE for the economy with ex-ante heterogeneous workers with the following properties for $i = 1, 2, \ldots, N$:

1. For all $(V, y) \in X \times Y$, $R(s_i, V, y) = R^i(V, y)$ and $m_i(s_i, V, y) = m^i(V, y)$.
2. For all $y \in Y$, $U(s_i, y) = U^i(y)$.
3. For all $(V, z, y) \in X \times Z \times Y$, $J(s_i, V, z, y) = J^i(V, z, y)$ and $c(s_i, V, z, y) = c^i(V, z, y)$.
4. For all $(V, y) \in X \times Y$, $\theta(m(s_i, V, y)) = \theta^i(m^i(V, y))$.

Implications

- The BRE with ex-ante heterogeneous workers is given by the endogenous stratification of the BRE of the N economies with homogeneous workers of type $i = 1, 2, \ldots, N$.
- To solve for the equilibrium with ex-ante heterogeneous workers, it is sufficient to solve for the equilibrium of the N economies with ex-ante homogeneous workers.
- Any active submarket is only visited by one type of worker.

Intuition

- Suppose that submarket $\mathbf{x} = (x_1, x_2)$ is visited by workers of type s_1 and s_2.
- Suppose that firm profits are greater if a firm fills a vacancy in submarket $\mathbf{x}$ with a worker of type s_1.
- Then, in submarket $\mathbf{x}' = (x_1, \underline{x})$, the expected profits of the firm are higher because no worker of type s_2 visits in $\mathbf{x}'$.
- Hence, $q(\theta(\mathbf{x}')) < q(\theta(\mathbf{x})) \Rightarrow \theta(\mathbf{x}') > \theta(\mathbf{x})$.
- Then a worker of type s_1 is strictly better of applying in market $\mathbf{x}'$.
- This contradicts our first assumption.