On Efficient Distribution with Private Information

by Atkeson and Lucas - RES 1992

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The model

- $t = 0, 1, 2, ...$
- Single, non storable consumption good, $y_t = y$
- Continuum of consumers
  - Preferences
$$E\left(\sum_{t=0}^{\infty}(1-\beta)\beta^{t}V(c_t)\theta_t\right)$$
$$V' > 0,\ V'' < 0,\ \theta_t \in \Theta = \{\theta_1, ..., \theta_n\}$$
$$\Pr(\theta_t = \theta_i) = \mu(\theta_i) \text{ for all } i = 1, ...n \text{ and all } t$$
  - Endowment: $w \in D \subset \mathbb{R}$
$$\Pr(w \in A) = \psi(A) \text{ for all } A \subseteq D$$
- θ_t is private information.

Planner's Objective

Minimize **constant** y to attain a certain distribution ψ **given the information available.**

Reporting Strategies

- **Reporting strategy**:

$$z = \left\{ z_t \left(\theta^t \right) \right\}_{t=0}^{\infty}$$

- **Truthful** reporting strategy: $z^* = \left\{ z_t^* \left(\theta^t \right) \right\}_{t=0}^{\infty}$

$$z_t^* \left(\theta^t \right) = \theta_t \ \ \forall t, \ \forall \theta^t \in \Theta^{t+1}.$$

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$$z_t^* \left(\theta^t \right) = \theta_t \quad \forall t, \ \forall \theta^t \in \Theta^{t+1}.$$

- Reporting history: $z^t = \left\{ z_0 \left(\theta_0 \right), ..., z_t \left(\theta_t \right) \right\}$

Plan

- **Plan**: $u = \{u_t(w, z^t)\}_{t=0}^{\infty}$ such that

$$\lim_{t\to\infty} \beta^t \sum_{s=0}^{\infty} \beta^s u_{t+s}\left(w, z^{t+s}\right) \theta_{t+s} = 0$$

where $u_t(w, z^t) = V(c_t(w, z^t))$.

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- Expected discounted utility

$$U(w, u, z) \equiv E\left(\sum_{t=0}^{\infty} (1-\beta)\,\beta^t u\left(w, z^t\right)\theta_t\right)$$

Allocation

A plan u is an **allocation** if it satisfies

(i) Truth Telling:

$$U(w,u,z^*) \geq U(w,u,z) \quad \text{for all } z \in Z \text{ and all } w \in D$$

(ii) Promise keeping:

$$U(w,u,z^*) = w \text{ for all } w$$

Planner's Problem

$$\min_{u \in A} y$$

such that

$$\int_{D \times \Theta} C\left(u_t\left(w, \theta^t\right)\right) d\mu\left(\theta_t\right) d\psi\left(w\right) \leq y \quad \text{for all } t$$

where $C\left(u_t\left(w, z^t\right)\right) \equiv c_t\left(w, z^t\right)$.

Allocation rule

A sequence $\sigma = \{f_t, g_t\}_{t=0}^{\infty}$ is an **allocation rule** if it satisfies

(i) Promise keeping: $\forall w_t$, $\forall\ t$

$$\int_{\Theta} \left[(1-\beta)\, f_t\,(w_t, \theta_t)\,\theta_t + \beta g_t\,(w_t, \theta_t) \right] d\mu = w_t$$

(ii) Truth telling: $\forall\ z_t \in \Theta$, $\forall w_t \in D$, $\forall t$

$$\left[(1-\beta)\, f_t\,(w_t, \theta_t)\,\theta_t + \beta g_t\,(w_t, \theta_t) \right] d\mu \geq \left[(1-\beta)\, f_t\,(w_t, z_t)\,\theta_t + \beta g_t\,(w_t, z_t) \right] d\mu$$

(iii) Boundedness: $\lim_{t\to\infty} \beta^t g_t \left(w_t(w_0, z^{*t-1}), z_t^* \right) = 0.$

Planner's Problem

$$\min_{\sigma \in R} y$$

such that for all t

$$\int_{D \times \Theta} C\left(f_t\left(w_t, \theta\right)\right) d\mu\left(\theta\right) d\psi_t\left(w\right) \leq y$$

where

$$\psi_{t+1}\left(w\right) = S_g\left(\psi_t\right)\left(w\right) \equiv \int_{D \times \Theta} I\left\{w_t : g_t\left(w_t, \theta\right) = w\right\} d\mu d\psi_t.$$

Equivalence

- $R(y,\psi)$: be the set of allocations /allocation rules that attain ψ with resources y.
- Then,

$$
\begin{aligned}
u &\in R(y,\psi) \longrightarrow \sigma \in R(y,\psi) \\
\sigma &\in R(y,\psi) \longrightarrow u' = \left\{ f_t\left(w_t(w_0, z^{t-1}), z_t\right) \right\} \in R(y,\psi)
\end{aligned}
$$

where

$$g_t\left(w_t, z_t\right) = w_{t+1}$$

Bellman Equation

$\varphi^*(\psi)$: minimum cost of attaining a distribution ψ.

$$\varphi(\psi) = \inf_{f,g} \max \left\{ \int_{D \times \Theta} C(f(w,\theta))\, d\mu(\theta)\, d\psi(w), \varphi^*(S_g \psi) \right\}$$

Problem T

Lemma 4.1 φ^* is a fixed point of T where

$$(T\varphi)(\psi) = \inf_{f,g \in B} \max\left\{ \int_{D\times\Theta} C(f(w,\theta))\, d\mu(\theta)\, d\psi(w), \varphi(S_g \psi) \right\}$$

Problem T

Lemma 4.2 If there are functions φ_a, φ_c and φ such that $\forall \psi$

- $\varphi_c(\psi) < \varphi^*(\psi) < \varphi_a(\psi)$
- $\lim_{n\to\infty} T^n \varphi_a(\psi) = \lim_{n\to\infty} T^n \varphi_c(\psi) = \varphi(\psi)$

Then, $\varphi = \varphi^*$.

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- Candidates:
  - Autarky: $\varphi_a(\psi) = \int_D C(w)\, d\psi$
  - Complete insurance: $\varphi_c(\psi) = \int_{D\times\Theta} C(u_c(w,\theta))\, d\psi d\varphi$

Example: log utility

- $V(x) = \log(x)$, $C(u) = \exp(u)$
- Bounding functions
  - $\varphi_a(\psi) = \int_D \exp(w)\, d\psi$
  - $\varphi_c(\psi) = \alpha \int_D \exp(w)\, d\psi$ where $\alpha = \exp\{-E[\theta \log \theta]\}$
- **The degree of inequality grows without bound when resources are efficientrly allocated.**

- **Lemma 3.1** $\forall\ \psi \in M$, if $u \in A$ attains ψ with resources y, $\exists$ $\sigma \in R$ that attains ψ with resources y.

- **Lemma 3.2** $\forall\ \psi \in M$, if $\sigma \in R$ attains ψ with resources y and $u =$ is the utility plan generated by σ, then $u \in A$ and attains ψ with resources y.