“Sectoral Price Data and Models of Price Setting”

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Objectives of the paper:

- Establish stylized facts about the dynamics of sectoral prices by estimating a statistical model.
- Contrast the predictions of the rational-inattention pricing model with those of the Calvo and sticky-information models.
\[ \pi_{nt} = \pi^A_{nt} + \pi^S_{nt} \]
\[ \pi^A_{nt} = a_n(L)u_t \]
\[ \pi^S_{nt} = b_n(L)\nu_{nt} \]

\( u_t \sim N(0, 1) \) : “aggregate shock”

\( \nu_{nt} \sim N(0, 1) \) : pairwise independent, independent of \( u_t \), “sector-specific shock”
Fig. 1. The cross-section of the normalized impulse responses of sectoral price indexes.

Note: This figure shows the posterior density of the normalized impulse responses of sector-specific shocks (top panel) and to aggregate shocks (bottom panel). The posterior density takes into account variation across sectors and parameter uncertainty. The results reported in this figure are discussed in Section 4.
Fig. 2. The cross-section of the speed of response of sectoral price indexes to shock.

Note: This figure shows the posterior density of the speed of response of sectoral price indexes to sector-specific shocks (top panel) and to aggregate shocks (bottom panel). The posterior density takes into account variation across sectors and parameter uncertainty. The speed of response is defined in Section 4.
Stylized Fact (3)

\[ \text{corr}(\text{the speed of price response in sector } n \text{ to a shock of type } K, \text{ std}(\pi^K_{nt})) > 0 \]

for \( K = A \) and \( S \)

(FYI: \( \text{std}(\pi^K_{nt}) \gg \text{std}(\pi^A_{nt}) \) for most sectors)
Model: Environment

- Discrete Time, Infinite Horizon.
- A continuum of sectors of mass one, indexed by $n \in [0, 1]$.
- Within each sector, a continuum of firms of mass one, indexed by $i \in [0, 1]$. 
A partial equilibrium model of firms

- $C_{int} = \left( \frac{P_{int}}{P_{nt}} \right)^{-\theta} \left( \frac{P_{nt}}{P_t} \right)^{-\eta} C_t$, ($\theta > 1$ and $\eta > 1$)
- $Y_{int} = Z_{nt} L_{int}^{\alpha}$, ($\alpha \in (0, 1]$) and $Y_{int} = C_{int}$
- $C_t$ and $Z_{nt}$ are exogenous.
- $\bar{p}_{int}$: (log of) profit-maximizing price implied by $C_t$ and $Z_{nt}$
- $\bar{p}_{int} = \bar{p}_{int}^A + \bar{p}_{int}^S$
Model: Firms

- At $t = 0$, firm chooses precision of $\{s_{int}\}_{t=0}^{\infty}$ ("signal")
- At each $t \geq 1$, firm chooses $p_{int}$ to minimize $E[\Pi(\bar{p}_{int}) - \Pi(p_{int})|s_{in}^t]$
- $E[\Pi(\bar{p}_{int}) - \Pi(p_{int})|s_{in}^t] \approx ... = \text{const.} \times E[(\bar{p}_{int} - p_{int})^2|s_{in}^t]$
- $p_{int} = E[\bar{p}_{int}|s_{in}^t]$
Model: \( \{ \bar{p}_{int} \}_{t=0}^{\infty} \) and \( \{ s_{int} \}_{t=0}^{\infty} \)

\[
\bar{p}_{int} = \bar{p}^A_{int} + \bar{p}^S_{int}
\]

\[
\begin{pmatrix}
\bar{p}^A_{int} \\
\bar{p}^S_{int}
\end{pmatrix} = \begin{pmatrix}
\bar{p}^A_{int-1} \\
\bar{p}^S_{int-1}
\end{pmatrix} + \begin{pmatrix}
\sigma_A u_t \\
\sigma_S \nu_{nt}
\end{pmatrix}
\]

\( u_t \sim N(0, 1) \) & \( \nu_{nt} \sim N(0, 1) \)

\( u_t \), independent of \( \nu_{nt} \) for all \( n \) and \( s \)

\[
\begin{pmatrix}
s^A_{int} \\
s^S_{int}
\end{pmatrix} = \begin{pmatrix}
s^A_{int} \\
s^S_{int}
\end{pmatrix} + \begin{pmatrix}
\sigma_\epsilon \epsilon_{int} \\
\sigma_\psi \psi_{int}
\end{pmatrix}
\]

\( \epsilon_{int} \sim N(0, 1) \) & \( \psi_{int} \sim N(0, 1) \)

\( \epsilon_{int} \), independent of \( \psi_{jms} \) for all \( j, m, s \)
Model: Firm's Problem at $t = 0$

$$\min_{(\sigma_\epsilon, \sigma_\psi) \in \mathbb{R}^2_+} \quad E\left[\sum_{t=1}^{\infty} \beta^t (p_{int} - \bar{p}_{int})^2\right]$$

subject to

(i) $\{\bar{p}_{int}\}_{t=0}^\infty$, $\{s_{int}\}_{t=0}^\infty$

(ii) $p_{int} = E[\bar{p}_{int}|s_{int}^t] \quad \forall \quad t \geq 1$

(iii) $\left(\frac{H(\bar{p}_{int}^A|s_{in}^{t-1})}{\kappa_A} - H(\bar{p}_{int}^A|s_{in}^t)\right) + \left(\frac{H(\bar{p}_{int}^S|s_{in}^{t-1})}{\kappa_S} - H(\bar{p}_{int}^S|s_{in}^t)\right) \leq \kappa$ for all $t = 1, 2, ...$

(iv) $s_{in0}$ satisfies $\text{Var} \left( \begin{pmatrix} \bar{p}_{in1}^A \\ \bar{p}_{in1}^S \end{pmatrix} | s_{in0} \right) = \lim_{t \to \infty} \text{Var} \left( \begin{pmatrix} \bar{p}_{int}^A \\ \bar{p}_{int}^S \end{pmatrix} | s_{int-1} \right)$
Can rewrite the firm’s problem as

$$\min_{(\kappa_A, \kappa_S) \in \mathbb{R}^2_+} \frac{\beta}{1 - \beta} \left( \frac{\sigma_A^2}{2\kappa_A} - 1 + \frac{\sigma_S^2}{2\kappa_S} - 1 \right)$$

s.t. \( \kappa_A + \kappa_S \leq \kappa \)

$$\Rightarrow \frac{2^{\kappa_S} - 2^{-\kappa_S}}{2^{\kappa_A} - 2^{-\kappa_A}} = \frac{\sigma_S}{\sigma_A}$$
Model: Characterizing the Solution

\[ \sigma_S > \sigma_A \Rightarrow \kappa_S > \kappa_A \]

\[ \bar{p}_{nt} - p_{nt} = \sum_{j=0}^{\infty} \left( \frac{1}{2^{2\kappa_A}} \right)^{j+1} \sigma_A u_{t-j} + \sum_{j=0}^{\infty} \left( \frac{1}{2^{2\kappa_S}} \right)^{j+1} \sigma_S \nu_{t-j} \] \Rightarrow \text{stylized facts (1) and (3)}

The speed of response to a shock of type K as a function of \( \sigma_K \) is concave for \( k = S \) and \( A \) \Rightarrow \text{stylized fact (2)}
Impulse Responses to Sector–Specific Shocks ($\kappa = 1, \sigma_A = 1, \sigma_S = 4$)

Impulse Responses to Aggregate Shocks ($\kappa = 1, \sigma_A = 1, \sigma_S = 4$)
The Speed of Response to Sector–Specific Shocks ($\kappa = 1$, $\sigma_A = 1$, $\sigma_S \sim N(4,1)$)

The Speed of Response to Aggregate Shocks ($\kappa = 1$, $\sigma_A = 1$, $\sigma_S \sim N(4,1)$)