"Adverse Selection in Competitive Search Equilibrium"

by Guerrieri, Shimer and Wright
presented by Saki Bigio

October 1, 2009

- Standard Search: McCall (1970).
- Matching: Pissarides (1985):
  - Matching function $m(u, v)$: u workers, v vacancies

$$u\alpha_w = m(u, v) = v\alpha_e$$

  - Wage is determined by Bargaining:

$$w \in \arg\max [W(w) - U]^{\theta} [J(y - w) - V]^{1-\theta}$$

- Equilibrium Matching.
- Directed Search: Shimer (1996)
  - Cost k to post Job Offer.
  - Subset of offers $\bar{w}$.
  - Higher offers attract more workers, but reduce matching rate.
  - In equilibrium worker and firms are indifferent.
- Directed Search with Heterogeneity.

- Study a *directed search* with *adverse selection.*
- Heterogeneity is ex-ante.

- Measure 1 of Agents.
- π_i measure of types $i \in \mathbb{I} \equiv \{1, 2, ..., I\}$.
- Large number of Principals.
- Cost of posting Contract k.
- Compact nonempty set of actions $\mathbb{Y} \subset \mathbb{R}^{\mathbb{N}}$ endowed with $d(y, y')$ metric.
- $\mathbb{Y}$ all actions by both types and transfers.
- Value for principal to match with i and action y:

$$v_i(y) - k$$

- Value for the agent to match with i and action y:

$$u_i(y)$$

- v_i, u_i continuous.

- Types are private information.
- Revelation Principle, *Contract is a vector* :

$$C = \{y_1, y_2 ... y_I\}$$

- IC if:

$$u_i(y_i) \geq u_i(y_j) \text{ for } \forall i, j$$

- Let $\mathbb{C} \subset \mathbb{Y}$ set of IC contracts.

- Set of posted contracts.
- $\Theta(C)$ denotes v/u ratio at contract C.
  - $\Theta : \mathbb{C} \rightarrow [0, \infty]$
- $\gamma_i(C)$ measure of type i in contract C.
  - $\Gamma(C) = \{\gamma_1(C), \gamma_2(C) ..., \gamma_I(C)\} \in \Delta^I$.

- Matching Function: $m(u, v)$ homogeneous, and weakly increasing in both arguments.
  - Probability of agent finding job $m(1, v/u) = \mu(\Theta)$
  - $\mu(\Theta(C)) : [0, \infty] \rightarrow [0, 1]$.
- Probability of Principal's match:
  - $\eta(\Theta(C)) = \frac{v}{u} m(u/v, 1) = \Theta(C) \eta(\Theta(C))$.
  - $\eta(\Theta(C))$ probability of match.
  - $\eta(\Theta(C)) \gamma_i(\Theta(C))$ probability of finding a type i.
- $\eta(0) = \bar{\eta} > 0$.(w.o. frictions $= 1$)
- $\mu(\infty) = \bar{\mu} > 0$.(w.o. frictions $=1$)
- $\mu(0) = \eta(\infty) = 0$.

- Expected Utility for Principal:

$$\eta\left(\Theta\left(C\right)\right)\sum_{i=1}^{I}\gamma_i\left(C\right)v_i\left(y_i\right)-\kappa$$

- Expected Utility for Agent reporting j.

$$\mu\left(\Theta\left(C\right)\right)u_i\left(y_j\right)$$

- Reservation Utility is 0.

Sketch goes here.

Definition

A Competitive Search Equilibrium (CSE) is a vector of $\bar{U} = \{U_i\}_{i \in I} \in \mathbb{R}_+^I$, a measure λ on $\mathbb{C}$ with support $\bar{\mathbb{C}}$. A function $\Theta(C) : \mathbb{C} \longrightarrow [0, \infty]$, and a function $\Gamma(C) : \mathbb{C} \longrightarrow \Delta^I$ satisfying:

1. Principals Profit Maximization and Free Entry Condition:
   Any $C \in \mathbb{C}$ we have:

$$\eta(\Theta(C)) \sum_{i=1}^{I} \gamma_i(C) v_i(y_i) \leq \kappa$$

   with equality if $C \in \bar{\mathbb{C}}$.

2. Optimal Search by Types: let

$$\bar{U}_i = \max\left\{0, \max_{C' \in \bar{\mathbb{C}}} \mu(\Theta(C)) u_i(y_i)\right\}$$

   with $\bar{U}_i = 0$ if $\bar{\mathbb{C}} = \varnothing$.
   Moreover, any, $\mathbb{C} \in C$

$$\bar{U}_i \geq \mu(\Theta(C)) u_i\left(y_j\right)$$

   with equality if $\Theta(C) < \infty$ and $\gamma_i(C) > 0$. And if $u_i(y_i) < 0$, then either $\Theta(C) = \infty$ or $\gamma_i = 0$.

3. Market Clearing:

$$\int_{\bar{\mathbb{C}}} \frac{\gamma_i(C)}{\Theta(C)} \lambda(dC) \leq \pi_i$$

   with equality if $U_i > 0$.

- Not Posted Contracts. Not part of definition.
- Firms know that on margin, they don't affect contracts (i).
- Agents only search among most attractive contracts. (Restriction to single offer).
- (iii) no equality means that if at 0, types randomize between entry and no entry, to equate the market.

Definition

$$\bar{\mathbb{Y}}_i = \{y \in \mathbb{Y} | \bar{\eta} v_i(y) \geq \kappa \text{ and } u_i(y) \geq 0\}$$

and let:

$$\bar{\mathbb{Y}} \equiv \cup_i \bar{\mathbb{Y}}_i$$

Assumption (A1: Monotonicity)

Any $y \in \bar{\mathbb{Y}}$

$$v_1(y) \leq v_2(y) \leq ... \leq v_I(y)$$

Assumption (A2: Local Non-Satiation)

All $j, i \in \mathbb{I}$, $j \leq i$, $y \in \bar{\mathbb{Y}}$ *and every* $\varepsilon > 0$, *there exists* $y\prime \in B_\varepsilon(y)$ *such that* $v_i(y') > v_i(y)$ *and* $u_j(y') \leq u_j(y)$.

Assumption (Sorting Condition)

Finally, for any $i \in \mathbb{I}$, *and* $y \in \bar{\mathbb{Y}}_i$ *and* $\varepsilon > 0$, *there exists a* $y' \in B_\varepsilon(y)$ *such that* $u_j(y) > u_j(y)$, $j \geq i$, *and* $u_j(y) < u_j(y)$, $j \leq i$.

- Main Contribution: equivalence among sequence of problems and equilibrium.

Problem (P-i Problem:)

$$\begin{aligned} \bar{U}_i &= \max_{\theta \in [0,\infty], y \in Y} \mu(\theta) u_i(y) \\ \textit{subject to} &: \eta(\theta) v_i(y) \geq k \\ \textit{and} &: \mu(\theta) u_j(y) \leq \bar{U}_j \textit{ for all } j < i \end{aligned}$$

- The P Problem is the solution to (P-i) for all i:
- $I^* \subset I$, and three vectors: $\{U_i\}_{i \in I}$,$\{\theta_i\}_{i \in I^*}$ and $\{y_i\}_{i \in I^*}$ such that:
  1. Subset I^* where $\{U_i\}_{i \in I^*}$ strictly positive.
  2. Any i$\in I^*, (\theta_i, y_i)$ solves the $(P - i)$ problem, given $(\bar{U}_1, \bar{U}_2, ..., \bar{U}_{I-1})$ and $\bar{U}_i = \mu(\theta_i) u_i(y_i)$.
  3. Any $i \notin I^*, \bar{U}_i = 0$.

Proposition (1)

Take a solution to P. Then, there exists a competitive search equilibrium with:

$$\{\bar{U}, \lambda, \bar{C}, \Theta, \Gamma\} \text{ with } \bar{U} = \{U_i\}_{i\in I}, \bar{C} = \{C_i\}_{i\in I^*}$$

where:

$$C_i = (y_i, y_i, .., y_i), \Theta(C_i) = \theta_i, \text{ and } \gamma_i(C_i) = 1$$

Proposition (2)

Any CSE, will solve will solve P. Select $I^* \equiv \{i | U_i > 0\}$. *Then, there is a contract* $C_i \in \{y_1, y_2, .., y_i, .., y_I\} \in \bar{C}$ *with* $\theta_i \in \Theta(C_i)$, $\gamma_i(C_i) > 0$. *Then,* $I^*, \{U_i\}_{i\in I}, \{\theta_i\}_{i\in I^*}$ *and* $\{y_i\}_{i\in I^*}$ *solve* P.

Proposition (3)

Solutions exists and are unique.

Proposition (4)

If there are strict gains from trade for all agents, all agents get positive utility.

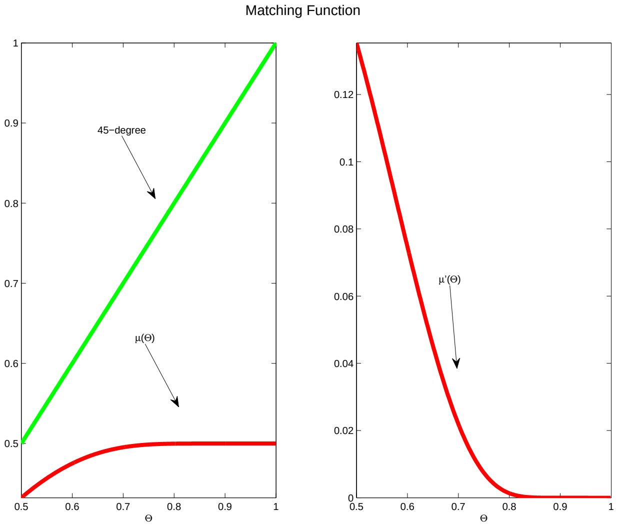
- Take

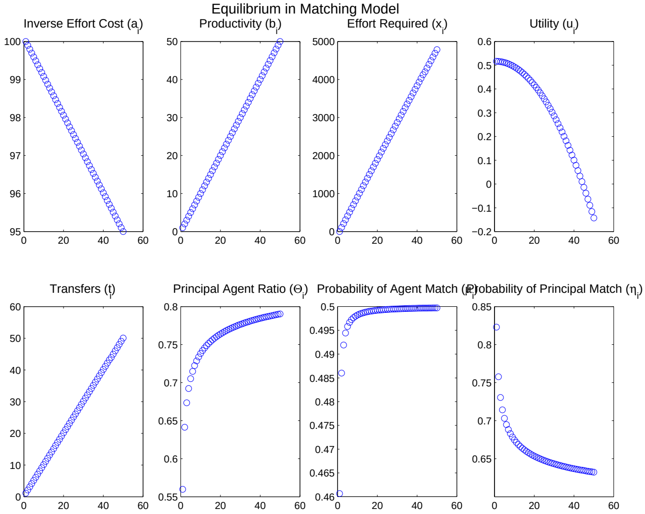
$$u_i = t - \frac{x}{a_i}$$

and the payoff for an unmatched type:

$$v_i = b_i - t$$

- I make: $I = \{1, 2, ..., 50\}$
- $\kappa = 0.1$


Matching Function
45−degree
μ(Θ)
μ'(Θ)
Θ
Θ
1
0.9
0.8
0.7
0.6
0.5
0.12
0.1
0.08
0.06
0.04
0.02
0
0.5 0.6 0.7 0.8 0.9 1


Equilibrium in Matching Model
Inverse Effort Cost (a_i)
Productivity (b_i)
Effort Required (x_i)
Utility (u_i)
Transfers (t_i)
Principal Agent Ratio (Θ_i)
Probability of Agent Match (μ_i)
Probability of Principal Match (η_i)

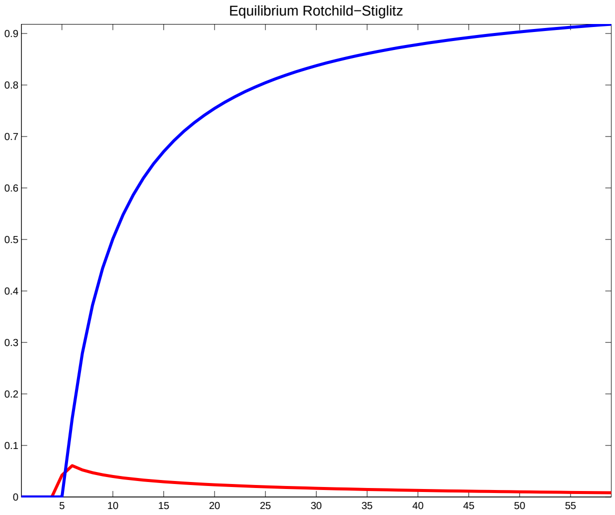
- Distortion from constrained Pareto optima.
- Pooling may be socially desirable.
- Under pooling, firms would deviate to screen agents.

Insurance Market:

$$u_i\left(c_e, c_u\right) = p_i U\left(c_e\right) + \left(1 - p_i\right) U\left(c_l\right)$$

and

$$v_i\left(c_e, c_u\right) = p_i U\left(1 - c_e\right) + \left(1 - p_i\right) U\left(1 - c_l\right)$$


Equilibrium Rotchild−Stiglitz
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5
10
15
20
25
30
35
40
45
50
55

- Framework differs from other models with Adverse Selection since: competition can eliminate pooling equilibria.
- Inefficiencies arise.
- Search Intensities is method for separation.
- Bad types can cause congestion, and reduce gains of trade with good types.