Understanding Predictability (JPE, 2004)
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Presented by Peter Gross

NYU

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Market returns forecastable by dividend yield
However, relation not stable (such as in the 1990s)
Price dividend ratio does not forecast dividend growth
Combination of time-varying risk tolerance and time-varying dividend growth rates can solve this problem
Increase in dividend growth leads to
- Increase in expected return (due to greater duration risk)
- Increase in price dividend ratio
This can weaken forecasting power of dividend yield for dividend growth and expected returns
Need to correct usual predictive regressions by adding in consumption price ratio
Overview

1. Model
2. Price Consumption Ratio
3. PD Ratios, Expected Returns, and Dividend Growth
4. Empirical Evaluation
   - Predictability of Dividend Growth
   - Predictability of Stock Returns
5. Conclusion
Model (1) - Agents

- Continuous Time, \( t \in [0, \infty) \).
- One perishable consumption good
- Representative agent has preferences given by:

\[
E \left[ \int_0^\infty e^{-\rho t} \log(C_t - X_t) \, dt \right]
\]

- \( C_t \) is current consumption, \( X_t \) external habit level
- Define the surplus ratio as

\[
S_t = \frac{C_t - X_t}{C_t}
\]
The inverse surplus ratio, \( Y_t = \frac{1}{S_t} \) and log consumption, \( c_t \) follow the processes

\[
\begin{align*}
    dY_t &= k(\bar{Y} - Y_t)dt - \alpha(Y_t - \lambda)(dc_t - E_t[dc_t]) \\
    dc_t &= \mu_c dt + \sigma_c dB_t^1
\end{align*}
\]

- \( \alpha > 0 \) ensures that positive innovations in consumption growth will lead to negative innovations in the inverse surplus
- \( \lambda \geq 1 \) ensures lower bound for inverse surplus ratio
- \( k \) controls spread of mean reversion
Model (3) - Cash Flow Process

- $n$ risky financial assets, paying $\{D_t^i\}_{i=1}^n$ units of consumption good at time $t$
- Other income cash flow $D_0^i$ (labor income, government transfers, ...)
- Consumption shares are given by $s_t^i = \frac{D_t^i}{C_t^i}$, $i = 1, 2, ..., n$ and governed by

\[
\begin{align*}
    ds_t^i &= \phi^i(\bar{s}_t^i - s_t^i)dt + s_t^i\sigma^i(s_t)dB_t^i \\
    \sigma^i(s_t) &= v^i - \sum_{j=0}^{n} s_t^j v^j
\end{align*}
\]

- Here, $B_t$ is a $N$-dimensional standard Brownian motion ($N \leq n + 1$), and $v^i$ is a vector of constants for each $i = 1, 2, ..., n$. 

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Log dividends, $\delta^i_t = \log(D^i_t)$, are governed by

$$d\delta^i_t = \mu^i_D(s_t)dt + \sigma_D(s_t)dB^i_t$$

$$\mu^i_D(s_t) = \mu_c + \phi^i \left( \frac{s^i}{s_t} - 1 \right) - \frac{1}{2} \sigma^i(s_t)\sigma(s_t)$$

$$\sigma_D(s_t) = \sigma_c + \sigma^i(s_t), \sigma_c = (\sigma_c, 0, 0, \ldots, 0)$$

Quantity of economic importance is $\text{Cov}(d\delta^i_t, dc_t) = \sigma_c^2 + \theta^i_{CF}$ where $\theta^i_{CF} = \nu^i_1\sigma_c$
Price/Consumption ratio of total wealth portfolio is given by

$$\frac{P_{t}^{TW}}{C_{t}} = \frac{1}{\rho} \left( \frac{\rho + k \bar{Y} S_{t}}{\rho + k} \right)$$

- If $S_{t}$ is at average, then everything collapses to standard case
- Decrease in risk aversion (increase in $S_{t}$) pushes ratio up
- The excess return is given by

$$dR_{t}^{TW} = \mu_{R}^{TW}(S_{t}) dt + \sigma_{R}^{TW}(S_{t}) dB_{t}^{1}, \text{ where}$$

$$\mu_{R}^{TW} = \left[ 1 + \alpha(1 - \lambda S_{t}) \right] \sigma_{R}^{TW}(S_{t})$$

$$\sigma_{R}^{TW}(S_{t}) = \left[ 1 + \frac{kYS_{t}(1 - \lambda S_{t})\alpha}{kYS_{t} + \rho} \right] \sigma_{c}$$
Fig. 1.—Aggregate quantities. 

- **Figure 1a**: Stationary density function of $S_p$. 
- **Figure 1b**: Price/consumption ratio of the total wealth portfolio. 
- **Figure 1c**: Expected excess returns and volatility of returns of the total wealth portfolio, and the risk-free rate. 
- **Figure 1d**: Sharpe ratio of the total wealth portfolio. 

The parameters used are those of table 1 below.
Without habit persistence, the price dividend ratio of \(i\)th asset is:

\[
\frac{P_t^i}{D_t^i} = \frac{1}{\rho + \phi^i} \left[ 1 + \frac{\phi^i \bar{\delta}^i}{\rho \bar{s}^i} \right]
\]

The expected excess return in turn is given by

\[
E_t[dR_t^i] = \sigma_c^2 + \frac{\theta^i_{CF} - \sum_{j=0}^{n} \theta^j_{CF} s_t^i}{1 + (\phi^i / \rho)(\bar{\delta}^i / \bar{s}^i)} \\
= b_0 + b_1^i(s_t) \frac{D_t^i}{P_t^i}
\]

Perfect linear relation between expected returns and dividend yield.

Increase in relative share increases price dividend ratio/expected return (more so if \(\phi^i / \rho\) is small)
PD Ratios and Expected Returns (2) - Habit

- With habits in place, no closed form solutions available
- However, very good approximations (error around 0.1 per cent) can be obtained:

\[
\frac{P_t^i}{D_t^i} \approx \hat{a}_0^i + \hat{a}_1^i S_t + \hat{a}_2^i \frac{\bar{s}^i}{s_t^i} + \hat{a}_3^i \bar{s}^i S_t
\]

\[
E_t[dR_t^i] \approx \hat{b}_0^i(S_t) + \hat{b}_1^i(S_t) \frac{D_t^i}{P_t^i} + \hat{b}_2^i(S_t) \frac{C_t}{P_t^i}
\]

\[
E_t[d\delta_t^i] \approx \hat{m}_0^i(S_t, s_t) + \hat{m}_1^i(S_t) \frac{P_t^i}{D_t^i}
\]

- Intuition for this: \( \frac{D_t^i}{P_t^i} \) more sensitive to relative share changes and \( \frac{C_t}{P_t^i} \) more sensitive to surplus consumption changes
Empirical Evaluation (1) - Data

- Quarterly dividends, returns, and market equity from CRSP
- Sample: 1947 - 2001
- 20 value-weighted industry portfolios
- Cash flows include both dividends and share repurchases
- Per capita consumption (NDS) from NIPA, deflated by PCE
Empirical Evaluation (2) - Calibration of Preferences

### TABLE 1
Model Parameters and Moments of Aggregate Quantities

**A. Preference Parameters and Consumption Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>.04</td>
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<tr>
<td>$\bar{Y}$</td>
<td>34</td>
</tr>
<tr>
<td>$k$</td>
<td>.16</td>
</tr>
<tr>
<td>$\lambda$</td>
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<tr>
<td>$\alpha$</td>
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<tr>
<td>$\mu_c$</td>
<td>.02</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>.01</td>
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</table>

**B. Aggregate Moments**

<table>
<thead>
<tr>
<th>Source</th>
<th>$E(R)$ (1)</th>
<th>Vol($R$) (2)</th>
<th>$E(\tau_f)$ (3)</th>
<th>Vol($\tau_f$) (4)</th>
<th>Ave(PC) (5)</th>
<th>Sharpe Ratio (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>.07</td>
<td>.16</td>
<td>.01</td>
<td>.01</td>
<td>30</td>
<td>.46</td>
</tr>
<tr>
<td>Model</td>
<td>.07</td>
<td>.23</td>
<td>.01</td>
<td>.04</td>
<td>30</td>
<td>.31</td>
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</tbody>
</table>
### C. SHARE PROCESS

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\beta^i$</th>
<th>$\phi^i$</th>
<th>$\theta^i \times 1000$</th>
<th>$\text{Cov}(\Delta c, \Delta \tilde{c})$</th>
<th>Horvath-Watson Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>.04</td>
<td>.52</td>
<td>-12</td>
<td>.05</td>
<td>14.26***</td>
</tr>
<tr>
<td>Railroads</td>
<td>.09</td>
<td>.20</td>
<td>-47</td>
<td>-.30</td>
<td>12.91**</td>
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<tr>
<td>Retail</td>
<td>.04</td>
<td>.20</td>
<td>00</td>
<td>.08</td>
<td>22.05*</td>
</tr>
<tr>
<td>Petroleum</td>
<td>.52</td>
<td>.16</td>
<td>-20</td>
<td>-.02</td>
<td>12.20**</td>
</tr>
<tr>
<td>Mining</td>
<td>.05</td>
<td>.16</td>
<td>-23</td>
<td>.15</td>
<td>3.98</td>
</tr>
<tr>
<td>Electrical equipment</td>
<td>.01</td>
<td>.12</td>
<td>-16</td>
<td>.01</td>
<td>5.18</td>
</tr>
<tr>
<td>Apparel</td>
<td>.12</td>
<td>.11</td>
<td>-10</td>
<td>.08</td>
<td>10.92*</td>
</tr>
<tr>
<td>Machinery</td>
<td>.05</td>
<td>.11</td>
<td>-10</td>
<td>-.02</td>
<td>14.13*</td>
</tr>
<tr>
<td>Paper</td>
<td>.01</td>
<td>.09</td>
<td>-10</td>
<td>.11</td>
<td>11.03**</td>
</tr>
<tr>
<td>Other transportation</td>
<td>.09</td>
<td>.09</td>
<td>-10</td>
<td>.15</td>
<td>14.83*</td>
</tr>
<tr>
<td>Transportation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>equipment</td>
<td>.25</td>
<td>.08</td>
<td>.27</td>
<td>.44</td>
<td>7.91</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>.05</td>
<td>.06</td>
<td>-13</td>
<td>.05</td>
<td>3.63</td>
</tr>
<tr>
<td>Other</td>
<td>.17</td>
<td>.06</td>
<td>-08</td>
<td>.09</td>
<td>11.85**</td>
</tr>
<tr>
<td>Fabricated metals</td>
<td>.03</td>
<td>.05</td>
<td>-17</td>
<td>.06</td>
<td>21.13*</td>
</tr>
<tr>
<td>Financial</td>
<td>.05</td>
<td>.04</td>
<td>-02</td>
<td>.15</td>
<td>27.30*</td>
</tr>
<tr>
<td>Chemical</td>
<td>.29</td>
<td>.03</td>
<td>-14</td>
<td>.05</td>
<td>9.33***</td>
</tr>
<tr>
<td>Primary metals</td>
<td>.12</td>
<td>.01</td>
<td>-32</td>
<td>-.14</td>
<td>1.19</td>
</tr>
<tr>
<td>Utilities</td>
<td>.10</td>
<td>.00</td>
<td>-06</td>
<td>.11</td>
<td>4.85</td>
</tr>
<tr>
<td>Food</td>
<td>.15</td>
<td>.00</td>
<td>-03</td>
<td>.09</td>
<td>5.30</td>
</tr>
<tr>
<td>Market portfolio</td>
<td>2.22</td>
<td>.07</td>
<td>-10</td>
<td>.07</td>
<td>15.70*</td>
</tr>
</tbody>
</table>

Note.—Panel A: Annualized preference and consumption process parameters chosen to calibrate the mean average excess returns, the average price/consumption ratio, the average risk-free rate and its volatility, and the Sharpe ratio of the market portfolio. Panel B: Expected excess return of the market portfolio, $E(R_t)$; standard deviation of return of the market portfolio, $\text{Vol}(R_t)$; standard deviation of the risk-free rate, $\text{Vol}(r)$, average price/consumption ratio, Ave(PC), and Sharpe ratio of the market portfolio. Panel C: Estimates of the long-run mean, $\beta$, and the speed of mean reversion $\phi$, cash flow risk $\theta$, and covariance between dividend growth and consumption growth, $\text{Cov}(\Delta d, \Delta c)$, for each industry. Industries are ordered, in this and subsequent tables, according to the parameter $\phi$. Col. 5 describes the likelihood ratio statistic for testing the null of no cointegration vs. the alternative of cointegration with the prespecified cointegrating vector as described in Horvath and Watson (1989). All entries in the table are in annual units.

* Those industries for which the null of no cointegration can be rejected at the 1 percent level.

** Those industries for which the null of no cointegration can be rejected at the 5 percent level.

*** Those industries for which the null of no cointegration can be rejected at the 10 percent level.
Cash flow predictive regressions:

\[
\Delta d_{t,t+\tau}^i = \beta^i_0 + \beta^i_X X_t + \epsilon_{t,t+\tau}^i, \quad \tau = 1, 4, \text{ or } 7 \text{ years}
\]

\[
X_t \text{ is } \frac{s^i_t}{s^i_t}, \frac{P^i_t}{D^i_t}, \text{ or both}
\]

• PD ratio is never significant when market dividends are used, and only weakly when industries are used.

• The relative share is a strong predictor of dividend growth, with $R^2$ up to 41 percent for market at the 7 year horizon.

• In individual regressions, relative share is significant predictor for 15 out of 20 industries, with $R^2$ above 30 percent in 10 cases.
Empirical Evaluation (5) - Predictability of Stock Returns

- Augmented expected return regression:

\[ r_{i,t+\tau}^i = \beta_0^i + \beta_D^i \frac{D_t^i}{P_t^i} + \beta_C^i \frac{C_t}{P_t^i} + \epsilon_{i,t+\tau}^i, \tau = 1, 4, \text{ or } 7 \text{ years} \]

- With market return, no significance for consumption price ratio, due to very slow mean reversion of market.

- With industry returns, the consumption price ratio becomes significant if mean reversion of dividends is quick.

- \( R^2 \) for some industries jumps dramatically (e.g., from 3 to 37 percent for mining) when this is included.
### Empirical Evaluation (6) - Relation between Dividends and Return Predictability

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**TABLE 7**

**SOURCE OF RETURN PREDICTABILITY**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>( \alpha_0 )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 ) (/100)</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Data: ( \beta_{D/P} ) vs. ( \phi^t ) and ( \theta^t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td>3.87 (.53)</td>
<td>-4.81 (1.74)</td>
<td>-15.36 (17.55)</td>
<td>.13</td>
</tr>
<tr>
<td>4-year</td>
<td>11.38 (1.62)</td>
<td>-16.14 (5.66)</td>
<td>10.13 (63.84)</td>
<td>.15</td>
</tr>
<tr>
<td>7-year</td>
<td>23.01 (3.02)</td>
<td>-35.05 (10.58)</td>
<td>55.40 (132.86)</td>
<td>.21</td>
</tr>
<tr>
<td></td>
<td>( \beta_{C/P} ) vs. ( \phi^t ) and ( \theta^t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td>.36 (.95)</td>
<td>7.78 (7.12)</td>
<td>-22 (36.28)</td>
<td>.04</td>
</tr>
<tr>
<td>4-year</td>
<td>2.76 (3.16)</td>
<td>24.04 (26.40)</td>
<td>-13.33 (111.26)</td>
<td>.04</td>
</tr>
<tr>
<td>7-year</td>
<td>7.26 (3.84)</td>
<td>28.04 (36.56)</td>
<td>-65.60 (159.60)</td>
<td>.03</td>
</tr>
<tr>
<td></td>
<td>B. Simulations: ( \beta_{D/P} ) vs. ( \phi^t ) and ( \theta^t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td>3.78 (.17)</td>
<td>-3.62 (.66)</td>
<td>5.19 (9.67)</td>
<td>.51</td>
</tr>
<tr>
<td>4-year</td>
<td>9.82 (.44)</td>
<td>-9.50 (1.75)</td>
<td>16.38 (25.35)</td>
<td>.50</td>
</tr>
<tr>
<td>7-year</td>
<td>11.26 (.51)</td>
<td>-10.92 (2.09)</td>
<td>18.85 (28.89)</td>
<td>.49</td>
</tr>
<tr>
<td></td>
<td>( \beta_{C/P} ) vs. ( \phi^t ) and ( \theta^t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-year</td>
<td>2.68 (.86)</td>
<td>3.15 (5.53)</td>
<td>23.91 (37.01)</td>
<td>.02</td>
</tr>
<tr>
<td>4-year</td>
<td>6.73 (2.18)</td>
<td>8.67 (14.55)</td>
<td>52.53 (95.15)</td>
<td>.02</td>
</tr>
<tr>
<td>7-year</td>
<td>7.87 (2.51)</td>
<td>9.96 (17.34)</td>
<td>52.68 (107.54)</td>
<td>.01</td>
</tr>
</tbody>
</table>

**Note.**—Panel A: The first panel reports the result of the cross-sectional regression of the predictive regression coefficient \( \beta_{D/P} \) (from Table 5) on the speed of mean reversion for the share, \( \phi^t \), and cash flow risk parameter, \( \theta^t \) (from Table 1). The second panel in panel A reports the same quantities but for the predictive regression coefficient \( \beta_{C/P} \). Here \( \beta_{D/P} \) and \( \beta_{C/P} \) are the regression coefficients in the return predictability regression

\[
\tau_{ukt} = \beta_0 + \beta_{D/P} \left( \frac{P_t}{P_{t+1}} \right) + \beta_{C/P} \left( \frac{P_t}{P_{t+1}} \right) + \epsilon_{ukt}, \quad \text{for } k = 1, 4, 7 \text{ years,}
\]

where the data are quarterly and the sample period is 1947–2001. Results are for the one-, four-, and seven-year horizons.

Panel B: Same as panel A but for simulated data, which consist of 40,000 quarters of simulated dividend and consumption data.
Conclusion

- An increase in dividend growth increases both expected returns and price dividend ratios.
- An increase in risk aversion increases expected returns while decreasing price dividend ratios.
- This makes dividend yield less reliable forecaster of future returns.
- By including consumption price ratio, one can decouple these two sources of return predictability.
- The extent to which dividends are predictable influences predictability of returns by the dividend yield.