Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence

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MAIN QUESTION

Can the observed Equity Premium be reconciled with a Stochastic Discount Factor (SDF) implied by households' optimizing behavior in each of the following settings?

1. An Incomplete Set of Markets to insure against idiosyncratic income shocks
2. Complete Markets
3. Complete Markets with Limited Participation in Asset Markets

A Test for SDF

Stochastic Discount Factor (SDF) is any m_t such that $\mathbb{E}_t[m_t R_t] = 1$

$$\Rightarrow \mathbb{E}[\quad m_t \quad \underbrace{(R_{M,t} - R_{F,t})}_{\text{Value Weighted Excess Market Return}} \quad] = 0$$

thus, to test the validity of a candidate SDF, compute the test statistic

$$u(m_t) = \frac{1}{T} \sum_{t=1}^{T} [m_t (R_{M,t} - R_{F,t})]$$

call this *The Unexplained Mean Premium* (given a certain SDF)

The Economy

- I households that participate in capital markets and trade J securities, each with total return $R_{j,t}$ between the dates $t-1$ and t.
- Preferences:

$$\mathbb{E}\left[(1-\alpha)^{-1}\sum_{i=0}^{\infty}\beta^t(c_{i,t}^{1-\alpha}-1)|F_0\right]$$

- In equilibrium, we get $I \times J$ Euler equations of consumption

$$\mathbb{E}_{t-1}(\underbrace{\beta g_{i,t}^{-\alpha}}_{m_t} R_{j,t}) = 1, \qquad g_{i,t} = \frac{c_{i,t}}{c_{i,t-1}}$$

- A complete set of markets for agents to insure against income risk $\Rightarrow g_{i,t} = g_t \; \forall i$

SDF with Incomplete Consumption Insurance

- How important is cross-sectional consumption growth variation?

- Can the lower moments of the cross-sectional consumption growth rate capture most of the cross-sectional variation?

- How important is the evolution of cross-sectional skewness of the consumption growth rate?

SDF with Incomplete Consumption Insurance

- How important is cross-sectional consumption growth variation?

$$m_t^1 = \beta I^{-1} \sum_{i=1}^{I} \left(\frac{c_{i,t}}{c_{i,t-1}} \right)^{-\alpha}$$

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- Can the lower moments of the cross-sectional consumption growth rate capture most of the cross-sectional variation?

$$g_t = I^{-1} \sum_{i=1}^{I} g_{i,t}$$

$$m_t^2 = \beta g_t^{-\alpha} \left[1 + \xi(\alpha) I^{-1} \sum_{i=1}^{I} \left(\frac{g_{i,t}}{g_t} - 1 \right)^2 - \psi(\alpha) I^{-1} \sum_{i=1}^{I} \left(\frac{g_{i,t}}{g_t} - 1 \right)^3 \right]$$

- How important is the evolution of cross-sectional skewness of the consumption growth rate?

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- How important is the evolution of cross-sectional skewness of the consumption growth rate?

$$m_t^3 = \beta g_t^{-\alpha} \left[1 + \xi(\alpha) I^{-1} \sum_{i=1}^{I} \left(\frac{g_{i,t}}{g_t} - 1 \right)^2 \right]$$

SDF with Complete Consumption Insurance

Consumption growth rate is equal across households: $g_{i,t} = g_t \forall i$

- SDF in terms of the cross-sectional mean of the household consumption growth rate

- Limited Participation in Asset Markets

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$$m_t^4 = \beta \left(I^{-1} \sum_{i=1}^{I} \frac{c_{i,t}}{c_{i,t-1}} \right)^{-\alpha} = \beta g_t^{-\alpha}$$

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- Limited Participation in Asset Markets

$$m_t^5 = \beta \left(\frac{\sum_{k \in K} c_{k,t}}{\sum_{k \in K} c_{k,t-1}} \right)^{-\alpha}$$

where K is the set of households with asset holdings above a certain (positive) threshold

THE EXERCISE

$$u(m_t(\alpha)) = \frac{1}{T}\sum_{t=1}^{T}\left[m_t(\alpha)(R_{M,t} - R_{F,t})\right]$$

- For each SDF $m_t^1, m_t^2, m_t^3, m_t^4, m_t^5$ compute u for a set of "plausible" RRA coefficients $\alpha \in 0, 1, 2, 3, 4, 5, ...$
- Report the mean unexplained premium for each setup and each RRA coefficient
- Test against the null that the computed mean unexplained premium is zero ($H_0 : u = 0$)
- Use household consumption data from the CEX, 1982-1996
- Time period is one quarter

Results I: Baseline

$$u = \frac{1}{T}\sum_{t=1}^{T}\left[\left(\beta I^{-1}\sum_{i=1}^{I}\left(\frac{c_{i,t}}{c_{i,t-1}}\right)^{-\alpha}\right)(R_{M,t} - R_{F,t})\right]$$

RRA (α)	0	1	2	3	4	5	6	7
Average u	1.85	1.95	2.32	1.88	$<$-10	$<$-10	$<$-10	$<$-10
p-value ($H_0 : u = 0$)	0.06	0.06	0.07	0.69	0.59	0.52	0.54	0.53

Note: The p-values are based on a bootstrap distribution

Results II: Which moments matter?

$$m_t^2 = \beta g_t^{-\alpha} \left[1 + \xi I^{-1} \sum_{i=1}^{I} \left(\frac{g_{i,t}}{g_t} - 1 \right)^2 - \psi I^{-1} \sum_{i=1}^{I} \left(\frac{g_{i,t}}{g_t} - 1 \right)^3 \right]$$

RRA (α)	0	1	2	3	4	5	6	7
	Mean, Variance and Skewness							
Average u	1.85	1.76	1.51	0.9	-0.22	-1.85	-4.12	-7.12
p-value	0.05	0.05	0.03	0.03	0.97	0.43	0.26	0.27
	Mean and Variance							
Average u	1.85	1.97	2.32	2.84	3.53	4.35	5.30	6.37
p-value	0.05	0.04	0.07	0.09	0.09	0.08	0.10	0.10

Results III: Complete Markets

$$m_t^4 = \beta \left(I^{-1} \sum_{i=1}^{I} \frac{c_{i,t}}{c_{i,t-1}} \right)^{-\alpha} = \beta g_t^{-\alpha}$$

RRA (α)	0	1	2	3	4	5	6	7
	”Vanilla” Complete Markets							
Average u	1.85	1.73	1.62	1.53	1.46	1.39	1.33	1.28
p-value ($H_0 : u = 0$)	0.05	0.04	0.06	0.07	0.04	0.04	0.06	0.06

RESULTS IV: CM + LIMITED PARTICIPATION

RRA (α)	0	1	5	10	15	20
	$a_{t-1} \geq \$10,000$					
Average u	5.18	4.89	3.97	3.07	2.09	0.51
p-value	0.00	0.00	0.03	0.47	0.83	0.93
	$a_{t-1} \geq \$20,000$					
Average u	5.18	4.82	3.65	2.02	-0.74	-5.80
p-value	0.03	0.00	0.05	0.80	0.97	0.85

Results *very* sensitive to (1) threshold choice, (2) holding period and (3) choice of quarter

Conclusion

- Equity Premium can be explained with economically plausible values of RRA, if one admits a non-degenerate distribution of consumption growth rates in the economy.
- The lower moments capture most of the cross-sectional variations in this distribution, but its *skewness* seems to be essential.
- The assumption of complete consumption markets can be weakly reconciled with the equity premium, given a limited (enough) participation in asset markets.

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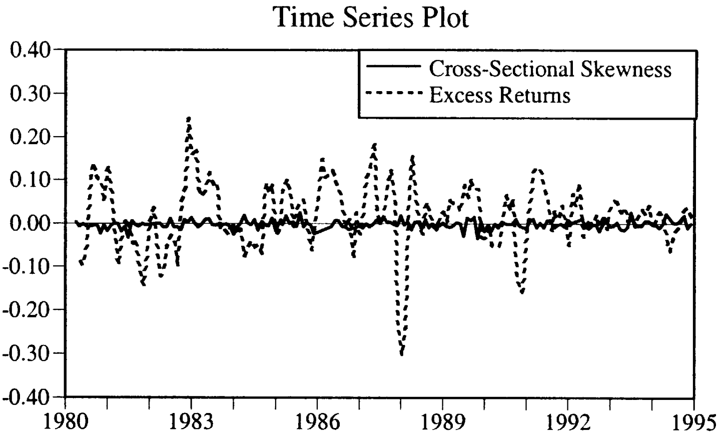

3RD ORDER EXPANSION WITH $\log(g_{i,t})$

- Recall: $m_t^1 = \beta I^{-1} \sum_{i=1}^{I} \left(\frac{c_{i,t}}{c_{i,t-1}}\right)^{-\alpha}$
- Taylor expansion in terms of $\log(g_{i,t}) = \log(c_{i,t}) - \log(c_{i,t-1})$

$$m_t^{\log} = \beta e^{-\alpha G_t} \left[1 + \frac{1}{2}\alpha^2 I^{-1} \sum_{i=1}^{I} (G_{i,t} - G_t)^2 - \frac{1}{6}\alpha^3 I^{-1} \sum_{i=1}^{I} (G_{i,t} - G_t)^3\right]$$

- This SDF fails; authors do not report results.

I.I.D. LOGNORMAL INCOME SHOCKS

- Assume that idiosyncratic income shocks are multiplicative and iid lognormal. From Constantinides and Duffie (1996), m_t^1 becomes:

$$m_t^{lnN} = \beta \left(\frac{\sum_{i=1}^{I} c_{i,t}}{\sum_{i=1}^{I} c_{i,t-1}} \right)^{-\alpha} \exp \left[\xi I^{-1} \sum_{i=1}^{I} \left(\log(g_{i,t}) - I^{-1} \sum_{i=1}^{I} \log(g_{i,t}) \right)^2 \right]$$

- This SDF fails; moreover, u increases as RRA increases.