Aggregate Consequences of Limited Contract Enforceability

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Summary

Goal: show that financial constraints (that arise from limited contract enforceability) have important aggregate consequences.

Approach: embed an optimal long-term contract between an entrepreneur and a financial intermediary into a general equilibrium model.

Key assumption: no market exclusion (entrepreneurs can default and still have access to the financial market).

Results: limited enforceability

1. amplifies the impact of transitory technological shocks
2. delays the aggregate impact of persistent technological shocks

Model - Preferences and Skills

Two types of agents: entrepreneurs (with mass one) and workers (with mass m)

Workers:

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t [c_t - \varphi(l_t)]$$

$$\Rightarrow \varphi'(l_t) = w_t \text{ (supply of labor)}$$

Entrepreneurs: die with probability α

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1-\alpha}{1+r}\right)^t c_t$$

Model - Technology and Shocks

Each entrepreneur runs a single project that:

1. needs an initial fixed setup cost: $I_0 > 0$
2. generates gross revenue: $zf(k, l)$

$z \in \{z_L, z_H\}$ is a project-specific productivity.

Let:

- N_t: number of available z_H-projects
- S_t: number of searching entrepreneurs
- $p_t = \max\{N_t/S_t, 1\}$: probability $z = z_H$.

N_t follows some stochastic process with prob. distribution $\Gamma(N_t; N_{t-1})$.

Model - Financial Contract and Repudiation

Assumptions:

1. $z_H - z_L$ is small enough such that surviving entrepreneurs are not willing to pay I_0 again to replace the project
2. if inactive, an entrepreneur looses his managerial ability, so a newborn entrepreneur always undertake the project (z_L or z_H)

Newborn entrepreneurs sign a long-term contract, specifying transfers, labor input and capital advancement, with a financial intermediary.

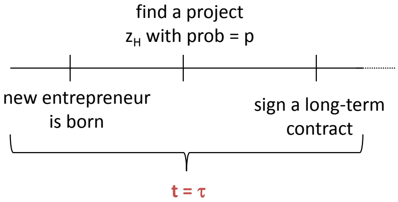
Intermediary commit, but entrepreneurs can default.

The value of default is:

$$D(k_{t-1}, \mathbf{s}_t) = k_{t-1} + \bar{V}(\mathbf{s}_t) - \kappa$$

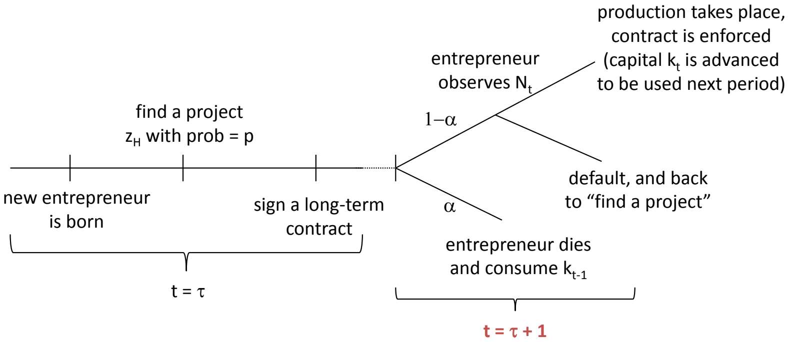
Model - Timing

Time line: at $\mathbf{t = \tau}$



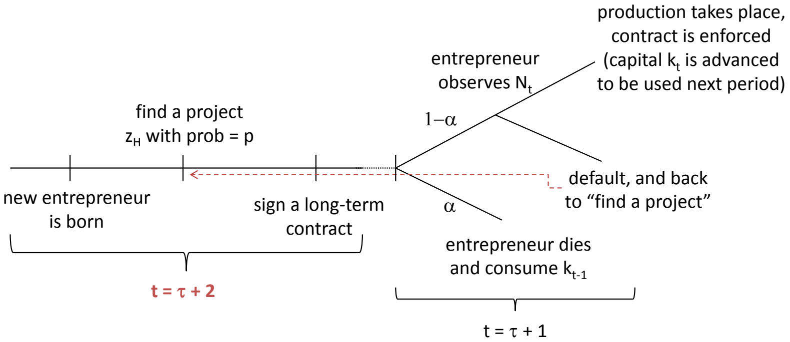
Model - Timing

Time line: at $\mathbf{t = \tau + 1}$



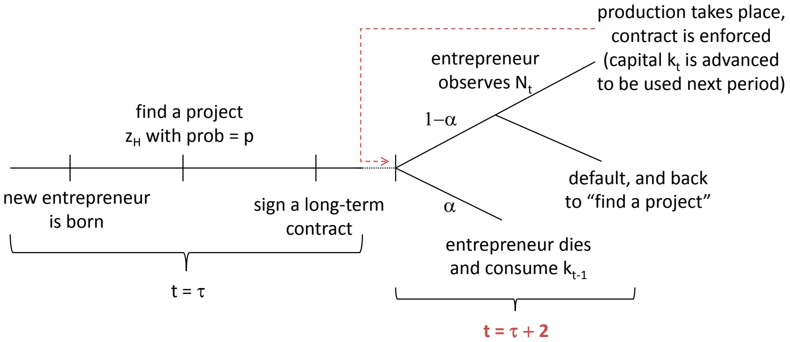
Model - Timing

Time line: at **$t = \tau + 2$, if default**



Model - Timing

Time line: at **$t = \tau + 2$, if NOT default**



Model - Firms' Expected Profits

Expected profits from period t investment

$$\pi(z; k_t, l_{t+1}, w_{t+1}) =$$
$$-k_t + \frac{1}{1+r}\{\alpha k_t + (1-\alpha)[(1-\delta)k_t + zf(k_t, l_{t+1}) - w_{t+1}l_{t+1}]\}$$

Optimal Financial Contract

$$V(z; \mathbf{s}_t) = \max_{\{d_\tau, k_\tau, l_{\tau+1}\}_{\tau=t}^{\infty}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} d_\tau$$

subject to

$$\gamma_{\tau+1}: \quad E_{\tau+1} \sum_{j=\tau+1}^{\infty} \beta^{j-\tau-1} d_j \geq D(k_\tau, \mathbf{s}_{\tau+1}) \text{ , for } \tau \geq t$$

$$\lambda_t: \quad E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\pi(z; k_\tau, l_{\tau+1}, w_{\tau+1}) - d_\tau\right] \geq l_0$$

$$d_\tau \geq 0$$

Recall: $D(k_\tau, \mathbf{s}_{\tau+1}) = k_\tau + \bar{V}(\mathbf{s}_{\tau+1}) - \kappa$

Optimal Financial Contract (saddle-point formulation)

$$\min_{\{\mu_{\tau+1}\}_{\tau=t}^{\infty}} \max_{\{d_\tau, k_\tau, l_{\tau+1}\}_{\tau=t}^{\infty}}$$

$$E_t \sum \beta^{\tau-t} \left[\pi(z; k_\tau, l_{\tau+1}, w_{\tau+1}) - (1-\mu_\tau)d_\tau - (\mu_{\tau+1} - \mu_\tau)\beta D(k_\tau, \mathbf{s}_{\tau+1})\right]$$

subject to

$$\mu_{\tau+1} = \mu_\tau + \frac{\gamma_{\tau+1}}{\lambda_t}$$

$$\mu_t = \frac{1}{\lambda_t}$$

$$d_\tau \geq 0$$

Proposition (Marcet and Marimon): a solution to the saddle-point problem is a solution to the original problem.

Optimal Financial Contract (saddle-point formulation)

Recursive formulation:

$$W(z; \mathbf{s}, \mu) = \min_{\mu(\mathbf{s}')} \max_{d,k,l'}$$

$$[\pi(z; k, l', w') - (1-\mu)d - \beta E(\mu(\mathbf{s}') - \mu)D(k, \mathbf{s}') + \beta EW(z; \mathbf{s}', \mu(\mathbf{s}'))]$$

subject to

$$\mu(\mathbf{s}') \geq \mu$$
$$d \geq 0$$
$$\mathbf{s}' \sim H(\mathbf{s})$$

Aggregate states: $\mathbf{s} = (N, \mathcal{M})$
where $\mathcal{M}(z, \mu)$ is measure of the firms over the variables z and μ,
and N is the number of high-productive projects.

Optimal Financial Contract (characterization)

First-order conditions:

$$\begin{aligned} d: &\quad \mu \le 1 \ (= \text{ if } d > 0) \\ k: &\quad \pi_k(z;k,l',w') = \beta E(\mu(\mathbf{s}') - \mu) \\ l': &\quad \pi_l(z;k,l',w') = 0 \\ \mu(\mathbf{s}'): &\quad D(k,\mathbf{s}') \le W_\mu(z;\mathbf{s}',\mu(\mathbf{s}')) \ (= \text{ if } \mu(\mathbf{s}') > \mu) \end{aligned}$$

Envelope condition:

$$W_\mu(z;\mathbf{s},\mu) = \begin{cases} d + \beta ED(k,\mathbf{s}') & \mathit{if}\, \mu(\mathbf{s}') > \mu \\ d + \beta EW_\mu(k,\mathbf{s}') & \mathit{if}\, \mu(\mathbf{s}') = \mu \end{cases}$$

Value of the contract for the entrepreneur: $W_\mu(z;\mathbf{s},\mu)$

$$W_\mu(z;\mathbf{s},\mu) = d + \beta EW_\mu(k,\mathbf{s}')$$

Optimal Financial Contract (properties)

Initial $\mu_t < 1$ (for a newborn entrepreneur at t) is such that it satisfies the zero profit condition for the intermediary.

If $\mu < 1$,

- $d = 0$ (no payments/consumption to the entrepreneur)
- $\mu(\mathbf{s}') > \mu$
- $D(k, \mathbf{s}') = W_\mu(z; \mathbf{s}', \mu(\mathbf{s}'))$ (enforc. constraint is binding)
- $\pi_k > 0$ (firm is financially constrained).

If $\mu = 1$,

- $d \geq 0$ (payments/consumption are not determined)
- $D(k, \mathbf{s}') < W_\mu(z; \mathbf{s}', \mu(\mathbf{s}'))$ (enforc. constraint ceases to bind)
- $\pi_k = 0$ (firm produces optimally - unconstrained).

Intuition: risk neutrality implies it is optimal to delay consumption (and increase promised values) so to relax the enforceability constraint ASAP.

General Equilibrium

Perform numerical exercises for three economies:

1. Limited enforcement without exclusion: $D(k_{-1}, \mathbf{s}) = k_{-1} + \bar{V}(\mathbf{s}) - \kappa$
2. Limited enforcement with exclusion: $D(k_{-1}, \mathbf{s}) = k_{-1} - \kappa$
3. Full enforcement: $D(k_{-1}, \mathbf{s}) = 0$

Recall: $p = \min\{N/S, 1\}$. In equilibrium, $S = \alpha$, and

$$\bar{V}(\mathbf{s}) = (1-p)V(z_L; \mathbf{s}) + pV(z_H; \mathbf{s})$$

Note: definition of recursive equilibrium is standard, except that we need to solve for a nontrivial fixed-point problem. Think of a map T such that

$$\bar{V}^{j+1}(\mathbf{s}) = T(\bar{V}^j)(\mathbf{s})$$

Numerically, it is another loop over $\bar{V}$.

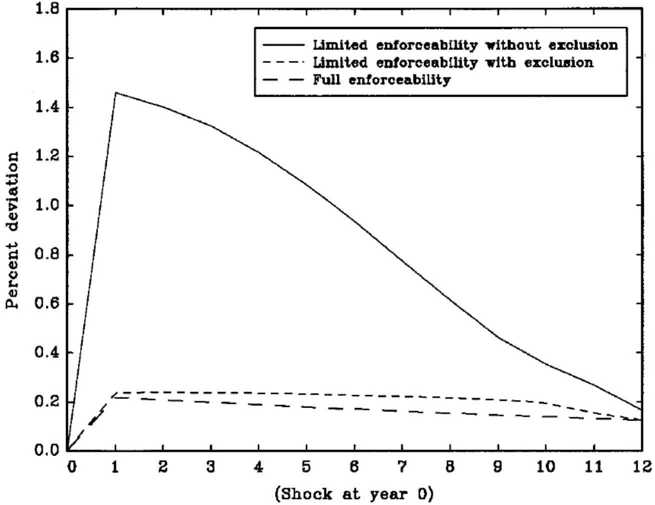
PARAMETER VALUES

Intertemporal discount rate	$r = .04$
Disutility from working $\varphi(l) \equiv A \cdot l^{(1+\epsilon)/\epsilon}$	$A = .001,\ \epsilon = 1$
Entrepreneurs' probability of death	$\alpha = .05$
Production technology $\bar{z} \cdot (k^{\nu} l^{1-\nu})^{\theta}$	$\bar{z} = .012,\ \theta = .85,\ \nu = .294$
Depreciation rate	$\delta = .0579$
Setup investment	$I_0 = .2$
Cost of repudiation	$\kappa = .35$

Result (amplification): output response to a temp. shock

Shock: p increases from 0.5 to 1 only for one period

$z_H - z_L$ is such that unconstrained z_H-projects are 30% larger



Result (amplification): output response to a temp. shock

Two effects of a temporary increase of z_H-projects:

(1) produc. of new firms $\uparrow \Rightarrow$ wage $\uparrow \Rightarrow$ old firms' scale $\downarrow$ and produc. $\uparrow$

(2) value of search $\uparrow \Rightarrow$ repudiation value $\uparrow \Rightarrow$ entrep. promised value $\uparrow$
$\Rightarrow$ more capital is advanced (financial constraint is less tight)

Second effect is absent in economies with full enforcement
($D(k_{-1}, \mathbf{s}) = 0$) or market exclusion ($\bar{V}(\mathbf{s}) = 0$).

Recall:

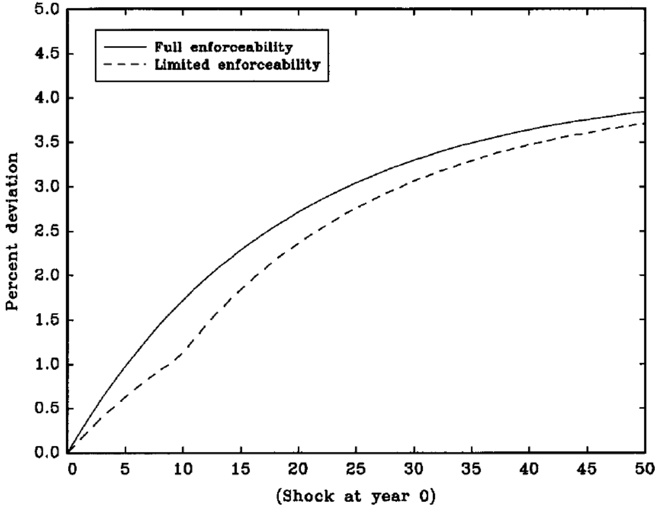
$$\bar{V}(\mathbf{s}) = (1-p)V(z_L; \mathbf{s}) + pV(z_H; \mathbf{s})$$

$$D(k_{-1}, \mathbf{s}) = k_{-1} + \bar{V}(\mathbf{s}) - \kappa$$

$$D(k_{-1}, \mathbf{s}) \leq W_\mu(z; \mathbf{s}; \mu)$$

Result (delaying diffusion): output resp. to a perm. shock

Shock (technology arrival): $N = 0$ ($p = 0$) increases to $N = S = \alpha$ ($p = 1$) permanently.



Result (delaying diffusion): output resp. to a perm. shock

Permanent vs. transitory shocks: from first-order and envelope conditions, if a firm is financially constrained,

$$k_{-1} + \bar{V}(\mathbf{s}) - \kappa = \beta[k + E\bar{V}(\mathbf{s}') - \kappa]$$

Intuition for sluggish diffusion:

- economies with full enforcement: all firms operate optimally
- economies with limited enforcement: new firms are initially small
- a new technology arrival increases output because new firms are more productive