Aggregate Consequences of Limited Contract Enforceability

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Journal of Political Economy, 2004
Summary

Goal: show that financial constraints (that arise from limited contract enforceability) have important aggregate consequences.

Approach: embed an optimal long-term contract between an entrepreneur and a financial intermediary into a general equilibrium model.

Key assumption: no market exclusion (entrepreneurs can default and still have access to the financial market).

Results: limited enforceability

1. amplifies the impact of transitory technological shocks
2. delays the aggregate impact of persistent technological shocks
Two types of agents: entrepreneurs (with mass one) and workers (with mass \( m \))

Workers:

\[
\max E_0 \sum_{t=0}^{\infty} \left( \frac{1}{1 + r} \right)^t [c_t - \varphi(l_t)]
\]

\[\Rightarrow \varphi'(l_t) = w_t \text{ (supply of labor)}\]

Entrepreneurs: die with probability \( \alpha \)

\[
\max E_0 \sum_{t=0}^{\infty} \left( \frac{1 - \alpha}{1 + r} \right)^t c_t
\]
Model - Technology and Shocks

Each entrepreneur runs a single project that:

1. needs an initial fixed setup cost: \( I_0 > 0 \)
2. generates gross revenue: \( zf(k, l) \)

\( z \in \{z_L, z_H\} \) is a project-specific productivity.

Let:

- \( N_t \): number of available \( z_H \)-projects
- \( S_t \): number of searching entrepreneurs
- \( p_t = \max\{N_t/S_t, 1\} \): probability \( z = z_H \).

\( N_t \) follows some stochastic process with prob. distribution \( \Gamma(N_t; N_{t-1}) \).
Assumptions:

1. $z_H - z_L$ is small enough such that surviving entrepreneurs are not willing to pay $I_0$ again to replace the project

2. If inactive, an entrepreneur loses his managerial ability, so a newborn entrepreneur always undertake the project ($z_L$ or $z_H$)

Newborn entrepreneurs sign a long-term contract, specifying transfers, labor input and capital advancement, with a financial intermediary.

Intermediary commit, but entrepreneurs can default.

The value of default is:

$$D(k_{t-1}, s_t) = k_{t-1} + \bar{V}(s_t) - \kappa$$
Model - Timing

Time line: at $t = \tau$

- Find a project $z_H$ with prob $= p$
- New entrepreneur is born
- Sign a long-term contract
- $t = \tau$
Time line: at $t = \tau + 1$

- New entrepreneur is born
- Sign a long-term contract
- Find a project $z_H$ with prob $= p$

If $t = \tau$:

- Entrepreneur dies and consume $k_{t-1}$
- Production takes place, contract is enforced (capital $k_t$ is advanced to be used next period)

If $1 - \alpha$:

- Default, and back to “find a project”

If $\alpha$:

- Entrepreneur observes $N_t$
Model - Timing

Time line: at $t = \tau + 2$, if default

- New entrepreneur is born
- Find a project $z_H$ with prob = $p$
- Sign a long-term contract
- Entrepreneur dies and consume $k_{t-1}$
- $t = \tau + 2$

Production takes place, contract is enforced (capital $k_t$ is advanced to be used next period)

Entrepreneur observes $N_t$

- Default, and back to “find a project”
- $t = \tau + 1$

$\alpha$

$1-\alpha$
Time line: at $t = \tau + 2$, if NOT default

- $t = \tau$:
  - new entrepreneur is born
  - find a project $z_H$ with prob $= p$

- $t = \tau + 2$:
  - sign a long-term contract
  - production takes place, contract is enforced (capital $k_t$ is advanced to be used next period)

- $1 - \alpha$:
  - entrepreneur observes $N_t$

- $\alpha$:
  - entrepreneur dies and consume $k_{t-1}$
  - default, and back to “find a project”

- $t = \tau + 2$:
  - new entrepreneur is born
  - find a project $z_H$ with prob $= p$

- If default:
  - Back to “find a project”
Expected profits from period $t$ investment

$$\pi(z; k_t, l_{t+1}, w_{t+1}) =$$

$$-k_t + \frac{1}{1+r}\{\alpha k_t + (1-\alpha)[(1-\delta)k_t + zf(k_t, l_{t+1}) - w_{t+1}l_{t+1}]\}$$
Optimal Financial Contract

\[ V(z; s_t) = \max_{\{d_\tau, k_\tau, l_{\tau+1}\}_{\tau=t}^\infty} E_t \sum_{\tau=t}^\infty \beta^{\tau-t} d_\tau \]

subject to

\[ \gamma_{\tau+1} : \quad E_{\tau+1} \sum_{j=\tau+1}^\infty \beta^{j-\tau-1} d_j \geq D(k_\tau, s_{\tau+1}) \text{, for } \tau \geq t \]

\[ \lambda_t : \quad E_t \sum_{\tau=t}^\infty \beta^{\tau-t} [\pi(z; k_\tau, l_{\tau+1}, w_{\tau+1}) - d_\tau] \geq l_0 \]

\[ d_\tau \geq 0 \]

Recall: \( D(k_\tau, s_{\tau+1}) = k_\tau + \bar{V}(s_{\tau+1}) - \kappa \)
Optimal Financial Contract (saddle-point formulation)

\[
\min_{\{\mu_{\tau+1}\}_{\tau=t}^{\infty}} \max_{\{d_{\tau}, k_{\tau}, l_{\tau+1}\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[ \pi(z; k_{\tau}, l_{\tau+1}, w_{\tau+1}) - (1 - \mu_{\tau}) d_{\tau} - (\mu_{\tau+1} - \mu_{\tau}) \beta D(k_{\tau}, s_{\tau+1}) \right]
\]

subject to

\[
\mu_{\tau+1} = \mu_{\tau} + \frac{\gamma_{\tau+1}}{\lambda_t}
\]

\[
\mu_t = \frac{1}{\lambda_t}
\]

\[
d_{\tau} \geq 0
\]

Proposition (Marcet and Marimon): a solution to the saddle-point problem is a solution to the original problem.
Optimal Financial Contract (saddle-point formulation)

Recursive formulation:

\[ W(z; s, \mu) = \min_{\mu(s')} \max_{d,k,l'} \left( \pi(z; k, l', w') - (1 - \mu)d - \beta E(\mu(s') - \mu)D(k, s') + \beta EW(z; s', \mu(s')) \right) \]

subject to

\[
\begin{align*}
\mu(s') &\geq \mu \\
 d &\geq 0 \\
 s' &\sim H(s)
\end{align*}
\]

Aggregate states: \( s = (N, M) \)

where \( M(z, \mu) \) is measure of the firms over the variables \( z \) and \( \mu \), and \( N \) is the number of high-productive projects.
Optimal Financial Contract (characterization)

First-order conditions:

\[ d : \quad \mu \leq 1 \quad (= \text{if } d > 0) \]
\[ k : \quad \pi_k(z; k, l', w') = \beta E(\mu(s') - \mu) \]
\[ l' : \quad \pi_l(z; k, l', w') = 0 \]
\[ \mu(s') : \quad D(k, s') \leq W_\mu(z; s', \mu(s')) \quad (= \text{if } \mu(s') > \mu) \]

Envelope condition:

\[
W_\mu(z; s, \mu) = \begin{cases} 
  d + \beta ED(k, s') & \text{if } \mu(s') > \mu \\
  d + \beta EW_\mu(k, s') & \text{if } \mu(s') = \mu 
\end{cases}
\]

Value of the contract for the entrepreneur: \( W_\mu(z; s, \mu) \)

\[
W_\mu(z; s, \mu) = d + \beta EW_\mu(k, s')
\]
Initial $\mu_t < 1$ (for a newborn entrepreneur at $t$) is such that it satisfies the zero profit condition for the intermediary.

If $\mu < 1$,

- $d = 0$ (no payments/consumption to the entrepreneur)
- $\mu(s') > \mu$
- $D(k, s') = W_{\mu}(z; s', \mu(s'))$ (enforc. constraint is binding)
- $\pi_k > 0$ (firm is financially constrained).

If $\mu = 1$,

- $d \geq 0$ (payments/consumption are not determined)
- $D(k, s') < W_{\mu}(z; s', \mu(s'))$ (enforc. constraint ceases to bind)
- $\pi_k = 0$ (firm produces optimally - unconstrained).

Intuition: risk neutrality implies it is optimal to delay consumption (and increase promised values) so to relax the enforceability constraint ASAP.
Perform numerical exercises for three economies:

1. Limited enforcement without exclusion: \( D(k_{-1}, s) = k_{-1} + \bar{V}(s) - \kappa \)
2. Limited enforcement with exclusion: \( D(k_{-1}, s) = k_{-1} - \kappa \)
3. Full enforcement: \( D(k_{-1}, s) = 0 \)

Recall: \( p = \min\{N/S, 1\} \). In equilibrium, \( S = \alpha \), and

\[
\bar{V}(s) = (1 - p)V(z_L; s) + pV(z_H; s)
\]

Note: definition of recursive equilibrium is standard, except that we need to solve for a nontrivial fixed-point problem. Think of a map \( T \) such that

\[
\bar{V}^{j+1}(s) = T(\bar{V}^j)(s)
\]

Numerically, it is another loop over \( \bar{V} \).
## Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intertemporal discount rate</td>
<td>( r = .04 )</td>
</tr>
<tr>
<td>Disutility from working ( \varphi(l) \equiv A \cdot l^{(1+\epsilon)/\epsilon} )</td>
<td>( A = .001, \ \epsilon = 1 )</td>
</tr>
<tr>
<td>Entrepreneurs’ probability of death</td>
<td>( \alpha = .05 )</td>
</tr>
<tr>
<td>Production technology ( \tilde{z} \cdot (k''l^{1-\nu})^\theta )</td>
<td>( \tilde{z} = .012, \ \theta = .85, \ \nu = .294 )</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta = .0579 )</td>
</tr>
<tr>
<td>Setup investment</td>
<td>( I_0 = .2 )</td>
</tr>
<tr>
<td>Cost of repudiation</td>
<td>( \kappa = .35 )</td>
</tr>
</tbody>
</table>
Result (amplification): output response to a temp. shock

Shock: \( p \) increases from 0.5 to 1 only for one period. 
\( z_H - z_L \) is such that unconstrained \( z_H \)-projects are 30% larger.
Two effects of a temporary increase of $z_H$-projects:

(1) produc. of new firms ↑ ⇒ wage ↑ ⇒ old firms’ scale ↓ and produc. ↑

(2) value of search ↑ ⇒ repudiation value ↑ ⇒ entrep. promised value ↑ ⇒ more capital is advanced (financial constraint is less tight)

Second effect is absent in economies with full enforcement ($D(k_{-1}, s) = 0$) or market exclusion ($\bar{V}(s) = 0$).

Recall:

$$\bar{V}(s) = (1 - p)V(z_L; s) + pV(z_H; s)$$

$$D(k_{-1}, s) = k_{-1} + \bar{V}(s) - \kappa$$

$$D(k_{-1}, s) \leq W_\mu(z; s; \mu)$$
Result (delaying diffusion): output resp. to a perm. shock

Shock (technology arrival): $N = 0 \ (p = 0)$ increases to $N = S = \alpha \ (p = 1)$ permanently.
Permanent vs. transitory shocks: from first-order and envelope conditions, if a firm is financially constrained,

\[ k_{-1} + \bar{V}(s) - \kappa = \beta [k + E\bar{V}(s') - \kappa] \]

Intuition for sluggish diffusion:

- economies with full enforcement: all firms operate optimally
- economies with limited enforcement: new firms are initially small
- a new technology arrival increases output because new firms are more productive