Expectations, Learning and Business Cycle Fluctuations

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This Paper:

- contrasts a standard RBC model under Rational Expectations with a RBC model that features the same functional forms for preferences and technology, but removes knowledge of the aggregate law of motion from the agents
- agents behave as econometricians

The Model

- continuum of households and firms, each of size 1
- discrete time, infinite horizon

The Model - Households

$$\widehat{E}_t^j \sum_{T=t}^{\infty} \beta^{T-t}[\ln C_T^j - v(H_t^j)] \tag{1}$$

$$C_t^j + K_{t+1}^j = R_t^K K_t^j + W_t H_t^j + (1-\delta)K_t^j \tag{2}$$

- $\widehat{E}_t^j$ subjective expectations operator for each agent j
- will be the same across agents in equilibrium, but agents do not know that
- *agents do not know law of motion for W_t, R_t^K and the aggregate capital stock*

The Model - Firms

$$Y_t^i = (K_t^i)^\alpha (X_t H_t^i)^{1-\alpha} \tag{3}$$

$$\Pi_t^i = Y_t^i - R_t^K K_t^i - W_t H_t^i \tag{4}$$

$$\ln(X_{t+1}/X_t) = \ln \overline{\gamma} + a_{t+1} \tag{5}$$

- $a_t \sim_{iid} N(0, \sigma_A^2)$

Notation

- variables without a superscript denote aggregate variables
- rescale variables to make them stationary (e.g. $c_t = C_t / X_t$)
- $\overline{variable}$ denotes the steady state value of *variable* and $\widehat{variable}_t$ its log deviation from steady state at time t (except for $\widehat{E}_t^j$)

Equilibrium/ Log-Linearization

- restrict attention to symmetric equilibria: $k_t^j = k_t$, $\widehat{E}_t^j = \widehat{E}_t$ ...
- log-linearized equilibrium condition for consumption (using the budget constraint and the Euler equation):

$$\widehat{c}_t = \frac{1-\beta}{\epsilon_c}[\beta^{-1}\widehat{k}_t - \beta^{-1}\widehat{\gamma}_t + \varepsilon_w \widehat{w}_t] \tag{6}$$

$$+\widehat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}[\frac{1-\beta}{\epsilon_c} - \beta]\beta\overline{R}\widehat{R^K}_{T+1} +$$

$$\widehat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\frac{1-\beta}{\epsilon_c}\beta\epsilon_w \widehat{w}_{T+1}$$

The Agents' Econometric Model

Each period the agents estimate a model of the following form:

$$\widehat{R^K}_t = \omega_0^r + \omega_1^r \widehat{k}_t + e_t^r \tag{7}$$

$$\widehat{w}_t = \omega_0^w + \omega_1^w \widehat{k}_t + e_t^w \tag{8}$$

$$\widehat{k}_{t+1} = \omega_0^k + \omega_1^k \widehat{k}_t + e_t^k \tag{9}$$

The Agents' Econometric Model

The agents estimate this model via recursive constant gain least squares:

$$\widetilde{\omega}_t = \widetilde{\omega}_{t-1} + gR_t^{-1}q_{t-1}(z_t - \widetilde{\omega}'_{t-1}q_{t-1}) \quad (10)$$

$$R_t = R_{t-1} + g(q_{t-1}q'_{t-1} - R_{t-1}) \quad (11)$$

where $z_t = (\widehat{R^K}_t \ \widehat{w}_t \ \widehat{k}_{t+1})$ and $q_{t-1} = (1 \ \widehat{k}_t)$

Forming Expectations

Once the parameters are estimated each period, the agents treat them as fixed (i.e. they don't take into account that the estimates might change in the future). They form expectations as follows:

$$\widehat{E}_t \widehat{R^K}_{t+1} = \widetilde{\omega}^r_{0,t-1} + \widetilde{\omega}^r_{1,t-1} \widehat{k}_{t+1} \tag{12}$$

It can then be shown that

$$z_t = T_1(\widetilde{\omega}_{t-1}) q_{t-1} + T_2(\widetilde{\omega}_{t-1}) \widehat{\gamma}_t \tag{13}$$

where T_1 and T_2 are *nonlinear* functions of ω

Calibration

Parameter Values are standard, except for σ_A and g, which are calibrated to minimize the quadratic distance between the volatility of HP-detrended output and the autocorrelation of output growth between the model and the data. σ_A for the RE model is chosen to match the volatility of HP-detrended output.

Table 1: HP filtered moments

	Statistic	Data	REE	Learning
Technology:	σ_A	-	1.22	0.98
Output:	σ_Y	1.54	1.54	1.52
Consumption:	σ_C/σ_Y	0.52	0.54	0.38
	$\rho_{Y,C}$	0.69	0.97	0.83
Investment:	σ_I/σ_Y	2.87	2.42	3.06
	$\rho_{Y,I}$	0.90	0.99	0.98
Hours:	σ_H/σ_Y	1.13	0.49	0.71
	$\rho_{Y,H}$	0.88	0.97	0.96
Wages:	σ_w/σ_Y	0.54	0.54	0.38
	$\rho_{Y,w}$	0.12	0.97	0.84
Labor Prod:	σ_{Pr}/σ_Y	0.68	0.54	0.38
	$\rho_{Y,Pr}$	0.52	0.97	0.84

Table 2: Growth rates

	Statistic	Data	REE	Learning
Output:	σ_{Δ_Y}	0.88	1.19	0.99
Consumption:	$\sigma_{\Delta_C}/\sigma_{\Delta_Y}$	0.60	0.52	0.54
	ρ_{Δ_Y,Δ_C}	0.51	0.98	0.80
Investment:	$\sigma_{\Delta_I}/\sigma_{\Delta_Y}$	2.54	2.45	2.82
	ρ_{Δ_Y,Δ_I}	0.71	0.99	0.94
Hours:	$\sigma_{\Delta_H}/\sigma_{\Delta_Y}$	0.93	0.50	0.65
	ρ_{Δ_Y,Δ_H}	0.70	0.98	0.87
Wages:	$\sigma_{\Delta_w}/\sigma_{\Delta_Y}$	0.60	0.52	0.54
	ρ_{Δ_Y,Δ_w}	0.08	0.98	0.80
Labor Prod:	$\sigma_{\Delta_{Pr}}/\sigma_{\Delta_Y}$	0.95	0.52	0.54
	$\rho_{\Delta_Y,\Delta_{Pr}}$	0.68	0.98	0.80

Impulse Response

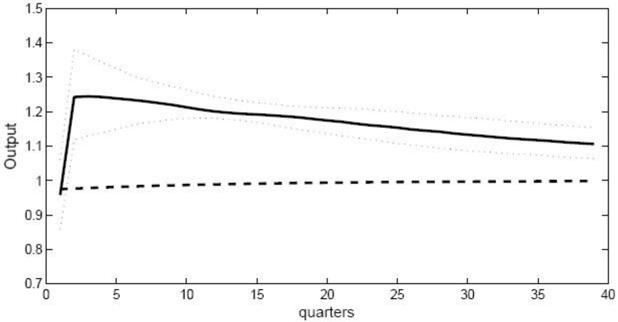

Figure 2: Dotted lines denote the 75% bands, solid line denotes the median impulse response under learning. The dashed line denotes the impulse response under rational expectations.

Impulse Response II

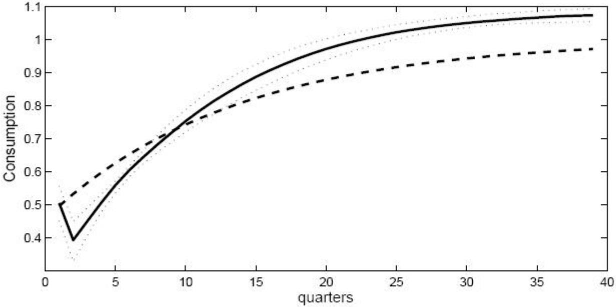

Figure 3: Dotted lines denote the 75% bands, solid line denotes the median impulse response under learning. The dashed line denotes the impulse response under rational expectations.

Impulse Response III

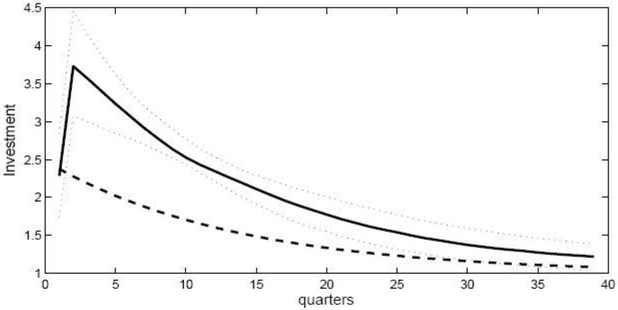

Figure 4: Dotted lines denote the 75% bands, solid line denotes the median impulse response under learning. The dashed line denotes the impulse response under rational expectations.

Impulse Response IV

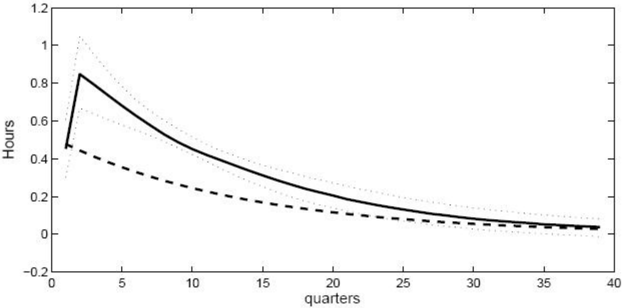

Figure 5: Dotted lines denote the 75% bands, solid line denotes the median impulse response under learning. The dashed line denotes the impulse response under rational expectations.

Distribution of Beliefs

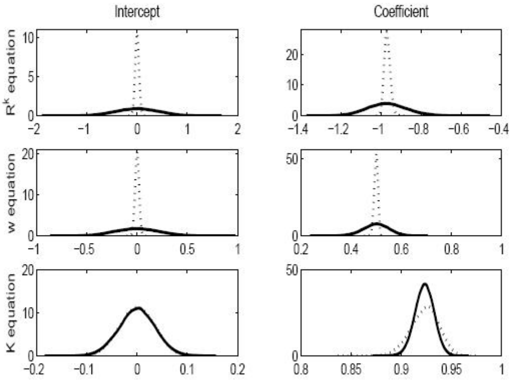

Figure 8: Solid line: model with feedback. Dotted line, model without feedback.