Individual Consumption Risk and the Welfare Cost of Business Cycles

Massimiliano De Santis
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Discussion by Christopher Tonetti
Question

- What are the welfare gains for consumers of removing aggregate consumption fluctuations?


- De Santis uses an economy with idiosyncratic consumption shocks and incomplete consumption insurance.
Role of Persistence

- Many early papers allowed only for transitory shocks.

- There is empirical evidence that consumers face permanent or highly persistent shocks to income.

- Without persistent shocks, consumers can almost fully insure themselves.

- If agents are almost fully insured, welfare gains from removing risk are small.
Baseline level of overall risk (aggregate and idiosyncratic) is important for calculating the welfare gain from removing a marginal unit of aggregate risk.

Welfare gain function is convex in level of overall consumption risk. What influences the degree of convexity?

Need to match the volatility and persistence of individual consumption to data.
Methodology

- Define experiment: What is the welfare gain from setting the aggregate component of consumption to its unconditional mean for all periods?
- Specify preferences and a consumption process.
- Construct a measure of welfare gain.
- Characterize welfare gain function for different parameterizations of consumption processes.
Environment

Preferences:

\[ E \left[ \sum_{t=0}^{\infty} \beta^t \frac{(C_t^i)^{1-\gamma}}{1 - \gamma} \right] \]  \hspace{1cm} (1)

Consumption:

\[
\ln C_t^i = \ln C_t + \ln \delta_t^i \]
\[
\ln C_{t+1} = \ln C_t + \mu + \sigma \eta_{t+1} \]
\[
\ln \delta_{t+1}^i = \ln \delta_t^i + \eta_{t+1}^i y_{t+1} - \frac{1}{2} y_{t+1}^2 \]
\[
y_{t+1}^2 = \bar{y}^2 + b \sigma \eta_{t+1} + \sigma_u u_{t+1} \]
\[
\eta_{t+1}, \eta_{t+1}^i, u_{t+1} \sim N(0, 1) \ i.i.d \]  \hspace{1cm} (5)

\[
\eta_{t+1}, \eta_{t+1}^i, u_{t+1} \sim N(0, 1) \ i.i.d \]  \hspace{1cm} (6)
Measuring the Welfare Gain

Define welfare gain $\Delta$ implicitly.

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t [(1 + \Delta)(C^i_t)]^{1-\gamma} \right] = E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\overline{C}^i_t)^{1-\gamma} \right]$$

- $C^i_t$ is the consumption stream in the economy with aggregate fluctuations.
- $\overline{C}^i_t$ is the consumption stream in the economy without aggregate fluctuations ($\eta_t = 0 \ \forall t$).
- In the economy without aggregate fluctuations
  $$\frac{\overline{C}_{t+1}}{\overline{C}_t} = e^{\mu + \frac{1}{2} \sigma^2}$$
Solving for Welfare Gain

Given the stochastic processes established we can compute $\Delta(\theta)$, where $\theta = (\beta, \gamma, \mu, \bar{y}^2, b, \sigma_\mu)$.

$$\frac{[(1 + \Delta)(C^i_0)]^{1-\gamma}}{1 - A(\theta)} = \frac{[(C^i_0)]^{1-\gamma}}{1 - A'(\theta)}$$

$$\Delta(\theta) = \left(\frac{1 - A'(\theta)}{1 - A(\theta)}\right)^{\frac{1}{\gamma-1}} - 1$$
Convexity

- $\Delta(\theta)$ is increasing in both $\bar{y}$ and $\sigma$.
- $\Delta(\theta)$ is convex in overall level of risk $\sigma$ and $\bar{y}$.
- $\Delta(\theta)$ is more convex in $\sigma$ when $\bar{y}$ is larger.
### Table 1. Parameter Choices

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean consumption growth (%)</td>
<td>$\mu$</td>
<td>1.89</td>
</tr>
<tr>
<td>Standard deviation of consumption growth (%)</td>
<td>$\sigma$</td>
<td>2.9</td>
</tr>
<tr>
<td>Mean idiosyncratic shock (%)</td>
<td>$\overline{\gamma}^2$</td>
<td>(10%)²</td>
</tr>
<tr>
<td>Std. Dev. idiosyncratic shock</td>
<td>$\sigma_u$</td>
<td>0.00389</td>
</tr>
<tr>
<td>Covariation with aggregate risk</td>
<td>$b$</td>
<td>0</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma$</td>
<td>2.4</td>
</tr>
<tr>
<td>Implied log risk-free rate (%)</td>
<td>$r^f$</td>
<td>1.4</td>
</tr>
<tr>
<td>Subjective discount factor*</td>
<td>$\beta$</td>
<td>0.99, 0.95</td>
</tr>
</tbody>
</table>
Figure 1. Individual Risk and Welfare Gains

\[ \Delta (\bar{y}^2 = 0.1^2, b = -0.3) \]

Gain from removing 10% of aggregate risk = 2.6%

\[ \Delta (\bar{y}^2 = 0.1^2, b = 0) \]

Gain = 1%

\[ \Delta (\bar{y}^2 = 0, b = 0) \]

Gain = 0.3%

\[ \frac{1}{2} \gamma \sigma^2 \equiv \text{Lucas' function} \]
Trend Stationary

What if $C_t$ was trend stationary?

$$\ln C_t = \delta + \mu t + \sigma \eta_t$$

**Figure 2.** Welfare Gains with Trend Stationary Per Capita Consumption

- $\Delta_T(\overline{y}^2, b = -0.3)$
- $\Delta_T(\overline{y}^2, b = -0.5)$
- $\Delta_T(\overline{y}^2 = 0.1^2)$
- $\frac{1}{2} \gamma \sigma^2 \equiv$ Lucas’ function

Gains from Removing Aggregate Risk

Aggregate Risk ($\sigma$)
In most of literature, idiosyncratic income shocks are not persistent. Imrohoroglu (89), KS(99,02)

Even granting individuals face significant consumption risk, how effective can macro policy be in reducing their risk?

Atkeson and Phelan (94) argue welfare gain from removing aggregate risk is zero. Lower aggregate risk is replaced with higher idiosyncratic risk.

Beuadry and Pages(01) argue removing aggregate fluctuations removes all of the persistent component of individual risk.

STY(2001) have persistence and countercyclical variances in idiosyncratic income process.
Conclusion

- The size of the welfare gain depends on the overall level of risk, not just the amount of risk policy removes.

- Persistence increases potential welfare gains.

- The welfare gain function is convex in aggregate risk $\sigma$ and idiosyncratic risk $\bar{y}^2$. 
Cross Sectional Standard Deviation

$y_t$ is cross sectional standard deviation of consumption growth.

\[
\frac{C_t^i}{C_{t-1}^i} = \frac{\delta_t^i}{\delta_{t-1}^i} \frac{C_t}{C_{t-1}} = \exp\{\eta_t^i y_t - .5 y_t^2\} \frac{C_t}{C_{t-1}}
\]

Conditioning on $C_t$

\[
\ln \left( \frac{C_t^i / C_t}{C_{t-1}^i / C_{t-1}} \right) = \eta_t^i y_t - .5 y_t^2 \sim N(-.5 y_t^2, y_t^2)
\]
Convexity

The convexity of the welfare gain can be seen from the equations for A and A'.

\[
A(\theta) = \beta \exp \left\{ \left[ (1 - \gamma)\mu + \alpha \bar{y}^2 \right] + \frac{1}{2} \left[ (1 - \gamma)\sigma + \alpha b\sigma \right]^2 + \frac{1}{2} \alpha^2 \sigma_u^2 \right\}
\]

\[
A'(\theta) = \beta \exp \left\{ \left[ (1 - \gamma)\left(\mu + \frac{1}{2}\sigma^2\right) + \alpha \bar{y}^2 \right] + \frac{1}{2} \alpha^2 \sigma_u^2 \right\}
\]

- A is increasing in both \( \bar{y} \) and \( \sigma \).
- When variability of aggregate consumption component, \( \eta \), is removed, the percentage change in utility (as A changes to A') will be larger when \( \bar{y} \) and \( \sigma \) are larger.