Individual Consumption Risk and the Welfare Cost of Business Cycles

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Question

- What are the welfare gains for consumers of removing aggregate consumption fluctuations?
- Lucas (1987) uses a representative agent framework with transitory shocks.
- De Santis uses an economy with idiosyncratic consumption shocks and incomplete consumption insurance.

Role of Persistence

- Many early papers allowed only for transitory shocks.
- There is empirical evidence that consumers face permanent or highly persistent shocks to income.
- Without persistent shocks, consumers can almost fully insure themselves.
- If agents are almost fully insured, welfare gains from removing risk are small.

Contribution

- Baseline level of overall risk (aggregate and idiosyncratic) is important for calculating the welfare gain from removing a marginal unit of aggregate risk.
- Welfare gain function is convex in level of overall consumption risk. What influences the degree of convexity?
- Need to match the volatility and persistence of individual consumption to data.

Methodology

- Define experiment: What is the welfare gain from setting the aggregate component of consumption to its unconditional mean for all periods?
- Specify preferences and a consumption process.
- Construct a measure of welfare gain.
- Characterize welfare gain function for different parameterizations of consumption processes.

Environment

Preferences:

$$E\left[\sum_{t=0}^{\infty}\beta^t\frac{(C_t^i)^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

Consumption:

$$\ln C_t^i = \ln C_t + \ln \delta_t^i \tag{2}$$

$$\ln C_{t+1} = \ln C_t + \mu + \sigma\eta_{t+1} \tag{3}$$

$$\ln \delta_{t+1}^i = \ln \delta_t^i + \eta_{t+1}^i y_{t+1} - \frac{1}{2}y_{t+1}^2 \tag{4}$$

$$y_{t+1}^2 = \overline{y}^2 + b\sigma\eta_{t+1} + \sigma_u u_{t+1} \tag{5}$$

$$\eta_{t+1},\ \eta_{t+1}^i,\ u_{t+1} \sim N(0,1)\ \ i.i.d \tag{6}$$

Measuring the Welfare Gain

Define welfare gain Δ implicitly.

$$E_0\left[\sum_{t=0}^{\infty}\beta^t[(1+\Delta)(C_t^i)]^{1-\gamma}\right]=E_0\left[\sum_{t=0}^{\infty}\beta^t(\overline{C}_t^i)^{1-\gamma}\right]$$

- C_t^i is the consumption stream in the economy with aggregate fluctuations.
- $\overline{C}_t^i$ is the consumption stream in the economy without aggregate fluctuations ($\eta_t = 0\ \forall t$).
- In the economy without aggregate fluctuations $\frac{\overline{C}_{t+1}}{\overline{C}_t} = e^{\mu+\frac{1}{2}\sigma^2}$

Solving for Welfare Gain

Given the stochastic processes established we can compute $\Delta(\theta)$, where $\theta = (\beta, \gamma, \mu, \bar{y}^2, b, \sigma_\mu)$.

$$\frac{[(1+\Delta)(C_0^i)]^{1-\gamma}}{1-A(\theta)} = \frac{[(C_0^i)]^{1-\gamma}}{1-A'(\theta)}$$

$$\Delta(\theta) = \left(\frac{1-A'(\theta)}{1-A(\theta)}\right)^{\frac{1}{\gamma-1}} - 1$$

Convexity

- $\Delta(\theta)$ is increasing in both $\overline{y}$ and σ.
- $\Delta(\theta)$ is convex in overall level of risk σ and $\overline{y}$.
- $\Delta(\theta)$ is more convex in σ when $\overline{y}$ is larger.

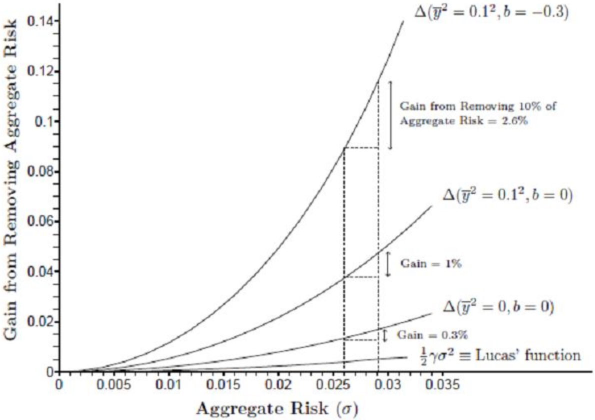
Calibration

TABLE 1. Parameter Choices

Parameter	Symbol	Value
Mean consumption growth (%)	μ	1.89
Standard deviation of consunption growth (%)	σ	2.9
Mean idiosyncratic shock (%)	$\overline{y}^2$	$(10\%)^2$
Std. Dev. idiosyncratic shock	σ_u	0.00389
Covariation with aggregate risk	b	0
Risk aversion	γ	2,4
Implied log risk-free rate (%)	r^f	1.4
Subjective discount factor*	β	0.99, 0.95

Main Result

FIGURE 1. Individual Risk and Welfare Gains

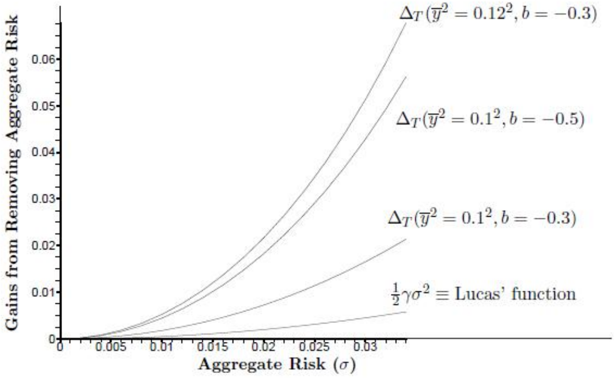


Trend Stationary

What if C_t was trend stationary?

$$\ln C_t = \delta + \mu t + \sigma \eta_t$$

FIGURE 2. Welfare Gains with Trend Stationary Per Capita Consumption



Existing Literature

- In most of literature, idiosyncratic income shocks are not persistent. Imrohoroglu (89), KS(99,02)
- Even granting individuals face significant consumption risk, how effective can macro policy be in reducing their risk?
- Atkeson and Phelan (94) argue welfare gain from removing aggregate risk is zero. Lower aggregate risk is replaced with higher idiosyncratic risk.
- Beuadry and Pages(01) argue removing aggregate fluctuations removes all of the persistent component of individual risk.
- STY(2001) have persistence and countercyclical variances in idiosyncratic income process.

Conclusion

- The size of the welfare gain depends on the overall level of risk, not just the amount of risk policy removes.
- Persistence increases potential welfare gains.
- The welfare gain function is convex in aggregate risk σ and idiosyncratic risk $\overline{y}^2$.

Cross Sectional Standard Deviation

y_t is cross sectional standard deviation of consumption growth.

$$\frac{C_t^i}{C_{t-1}^i} = \frac{\delta_t^i}{\delta_{t-1}^i} \frac{C_t}{C_{t-1}} = \exp\{\eta_t^i y_t - .5y_t^2\} \frac{C_t}{C_{t-1}}$$

Conditioning on C_t

$$\ln\left(\frac{C_t^i/C_t}{C_{t-1}^i/C_{t-1}}\right) = \eta_t^i y_t - .5y_t^2 \sim N(-.5y_t^2, y_t^2)$$

Convexity

The convexity of the welfare gain can be seen from the equations for A and A'.

$$A(\theta) = \beta \exp \left\{ [(1-\gamma)\mu + \alpha \overline{y}^2] + \frac{1}{2}[(1-\gamma)\sigma + \alpha b \sigma]^2 + \frac{1}{2}\alpha^2 \sigma_u^2 \right\}$$

$$A'(\theta) = \beta \exp \left\{ [(1-\gamma)(\mu + \frac{1}{2}\sigma^2) + \alpha \overline{y}^2] + \frac{1}{2}\alpha^2 \sigma_u^2 \right\}$$

- A is increasing in both $\overline{y}$ and σ.
- When variability of aggregate consumption component, η, is removed, the percentage change in utility (as A changes to A') will be larger when $\overline{y}$ and σ are larger.