A Theory of Financial Constraints and Firm Dynamics

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Model

- Discrete time, infinite horizon. $t = 0, 1, \ldots$
- 2 classes of risk neutral individuals with discount factor δ
  - Entrepreneurs/Borrowers: endowment $M > 0$, access to a risky project at $t = 0$.
  - Banks/Lenders: unlimitted endowment

Risky Project

- Initial investment at $t = 0$, $I_0 > M$
- Per period investment of working capital k_t
- Per period return depends on a random variable $\theta \in \Theta = \{L, H\}$ where $\Pr(\theta = H) = p$.

$$\text{Return} = \begin{cases} 0 & \text{if } \theta = L \\ R(k) > 0 & \text{if } \theta = H \end{cases}$$

  - Revenue outcome is private information for the entrepreneur
- Liquidation generates a scrap value $S > 0$

Contract

- Perfect enforcement environment.
- Take-it-or leave-it offer from the bank to the entrepreneur.
- Let $h^t = \left\{\widehat{\theta}_\tau\right\}_{\tau=0}^{t}$ be the history of reports $\widehat{\theta}_t$ up to period t.

Definition

A contract $\sigma = \left\{\alpha_t\left(h^{t-1}\right), Q_t\left(h^{t-1}\right), k_t\left(h^{t-1}\right), \tau_t\left(h^t\right)\right\}_{t=0}^{\infty}$ specifies:

- contingent liquidation probabilities $\alpha_t\left(h^{t-1}\right)$
- transfers Q_t from the lender to the borrower in case of liquidation
- capital advancements k_t
- payments τ_t from the borrower to the lender in case of no liquidation.

Feasible contract

Definition

A contract σ is feasible if $\forall t \geq 1$ and $\forall h^{t-1} \in \Theta^{t-1}$ we have

- $\alpha_t \left(h^{t-1}\right) \in [0, 1]$
- $Q_t \geq 0$
- $\tau_t \left(h^{t-1}, H\right) \leq R\left(k_t \left(h^{t-1}\right)\right)$
- $\tau_t \left(h^{t-1}, L\right) \leq 0$

Timing

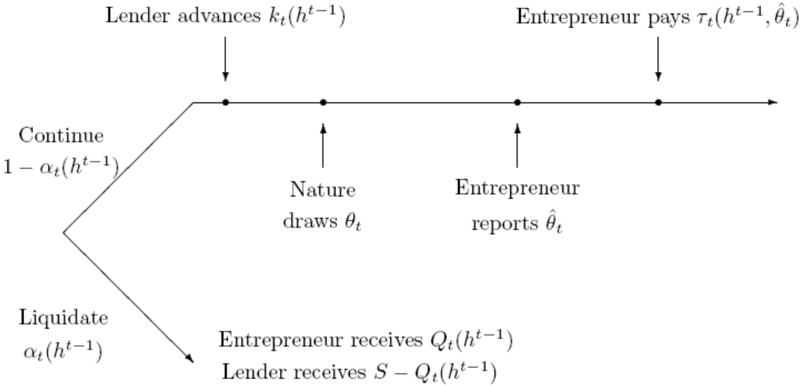

Figure 1: The Timing.

Value Functions

- Expected discounted cash flows from contract σ and reporting strategy $\widehat{\theta} = \left\{ \widehat{\theta}_t \left(\theta^t \right) \right\}_{t=1}^{\infty}$
  - for the entrepreneur: $V_t \left(\widehat{\theta}, \sigma, h^{t-1} \right) \Longrightarrow$ **Equity**
  - for the bank: $B_t \left(\widehat{\theta}, \sigma, h^{t-1} \right) \Longrightarrow$ **Debt**

Definition

A contract σ is **incentive compatible** (IC) if $\forall\ \widehat{\theta}$

$$V_1\left(\theta,\sigma,h^0\right)\geq\ V_1\left(\widehat{\theta},\sigma,h^0\right)$$

Optimal contract

$$\mathcal{V} = \left\{ V | \exists \sigma \text{ feasible and IC such that } V_1\left(\theta, \sigma, h^{t-1}\right) = V \right\}$$

Definition

For any $V \in \mathcal{V}$, an optimal contract is the feasible and incentive compatible contract that maximizes the value obtained by the lender while delivering an initial value V to the entrepreneur.

$$B\left(V\right) = \sup\left\{ B | \exists \sigma \text{ s.t. } V_1\left(\theta, \sigma, h^{t-1}\right) = V \text{ and } B_1\left(\theta, \sigma, h^{t-1}\right) = B \right\}$$

Capital structure $\left(V, B\left(V\right)\right)$ implies a value of the firm of $W\left(V\right) = B\left(V\right) + V$

Benchmark: Symmetric information

- Maximize total surplus from the match
  $\Longrightarrow$ Each period the bank lends the entrepreneur the uncostrained optimal level of working capital k^*
- Any division of surplus is feasible.
- Value of the firm is $\widetilde{W}$

Private information: Recursive Formulation

- The value of equity summarizes all information provided by the history itself
  $\Longrightarrow$ Recursive formulation with V as a state variable

$$V = p\left(R\left(k\right) - \tau\right) + \delta\left[pV^{H} + \left(1 - p\right)V^{L}\right]$$

- IC becomes

$$\begin{aligned} R\left(k\right) - \tau + \delta V^{H} &\geq R\left(k\right) + \delta V^{L} \\ \tau &\leq \delta\left(V^{H} - V^{L}\right) \end{aligned}$$

Firm Value

$\widehat{W}$: value of the firm contingent on no liquidation
W :value of the firm prior to the liquidation decision.

$$\widehat{W}(V) = \max_{k,\tau,V^H,V^L} [pR(k) - k] + \delta \left[pW\left(V^H\right) + (1-p)\,W\left(V^L\right) \right]$$

subject to

$$\begin{aligned}
V &= p\left(R(k) - \tau\right) + \delta\left[pV^H + (1-p)\,V^L\right] \\
\tau &\leq \delta\left(V^H - V^L\right) \\
\tau &\leq R(k) \\
V^H, V^L &\geq 0
\end{aligned}$$

Firm value prior to liquidation decision

$$W(V) = \max_{\alpha \in [0,1], Q, V_c} \alpha S + (1-\alpha)\widehat{W}(V_c)$$

subject to

$$\begin{aligned} \alpha Q + (1-\alpha) V_c &= V \\ Q, V_c &\geq 0 \end{aligned}$$

Firm value

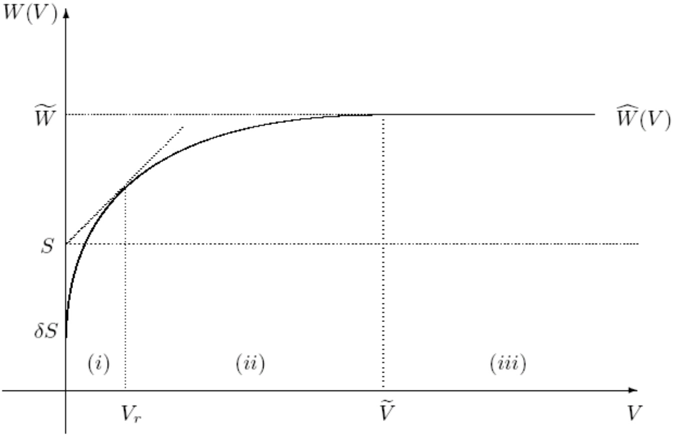

Figure 2: The Value Function.

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If $V \in \left[V_r, \widetilde{V}\right] \tau = R(k)$ and

$$W(V) = \max_{k,\tau,V^H,V^L} [pR(k) - k] + \delta \left[pW\left(V^H\right) + (1-p) W\left(V^L\right)\right]$$

subject to

$$\begin{aligned} V &= \delta \left[pV^H + (1-p) V^L\right] \\ R(k) &\leq \delta \left(V^H - V^L\right) \\ V^H, V^L &\geq 0 \end{aligned}$$

Trade off between increasing k and a mean preserving spread of expected value. (since W is concave)

Evolution of Equity

If $V_r < V < \widetilde{V}$, V_t is a submartingale with 2 absorbing sets, $V_t = 0$ and $V_t \geq \widetilde{V}$

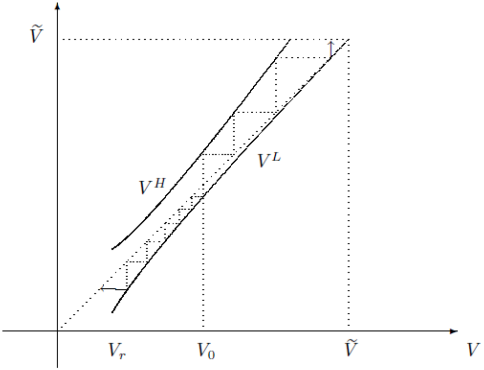

Figure 3: The Dynamics of Equity.

Firm dynamics

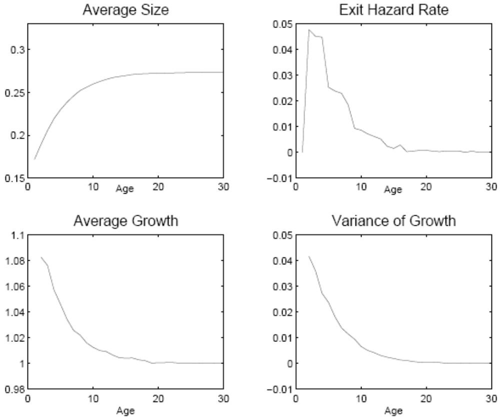

Figure 5: Dynamics of Growth and Survival.

Firm value prior to liquidation decision

- 3 regions
  - $V < V_r$ liquidation is possible. Lotery $Q = 0$ with prob $\alpha(V)$ and V_r otherwise
  - $V \in \left[V_r, \widetilde{V}\right)$, no liquidation, $k < k^*$. The entrepreneur is borrowing constaint. $\tau = R(k) = \delta\left(V^H - V^L\right)$(no dividends paid +Binding IC)
  - $V \geq \widetilde{V}$, no liquidation, $k = k^*$. Transfers are not determined.