Objective of the paper:

- Introduce a model with differences in liquidity across assets, where fiat money may have value in equilibrium
- Analyze the effects of aggregate productivity and liquidity shocks
- Study the role of open market operations where the government prints money to buy equity
Environment

4 objects traded:
- Nondurable general output
- Labor
- Equity
- Fiat money

2 types of agents:
- Entrepreneurs
- Workers
Preferences: \( \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \)

Access to production technology \( y_t = A_t k_t^\gamma l_t^{1-\gamma} \)
\( \rightarrow y_t - w_t l_t = r_t k_t \)

Investment opportunity (to produce new capital) arrives with probability \( \pi \), iid across agents and time
\( k_{t+1} = \lambda k_t + i_t \)
Financing investment: equity issuance and (possibly) fiat money
2 key assumptions:

- Entrepreneur cannot pledge more than fraction $\theta$ of future returns from new capital \([borrowing\ constraint]\)
- Entrepreneur can only sell up to fraction $\phi_t$ of equity \([resaleability\ constraint]\)

Simplifying assumption: resaleability constraint affects inside equity (own unmortgaged capital) as well as outside equity
Environment: Entrepreneurs

\[ n_t = \text{total equity} \]
\[ m_t = \text{fiat money holdings} \]
\[ s_t = \text{equity issued} \]

Constraints imply

- \[ n_{t+1} = \lambda n_t + i_t - s_t \]
- \[ s_t \leq \theta i_t + \phi_t \lambda n_t \]

Eliminate \( s_t \):

- \[ n_{t+1} \geq (1 - \theta) i_t + (1 - \phi_t) \lambda n_t \]; also
- \[ m_{t+1} \geq 0 \]
- \[ c_t + i_t + q_t(n_{t+1} - i_t - \lambda n_t) + p_t(m_{t+1} - m_t) = r_t n_t \]

where \( q_t \) is the price of equity and \( p_t \) the price of fiat money
Environment: Workers

Preferences: \( E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c'_s - \frac{\omega}{1+\nu} l'^{1+\nu}) \)

Constraints:
- \( c'_t + q_t(n'_{t+1} - \lambda n'_t) + p_t(m'_{t+1} - m'_t) = w_t l'_t + r_t n'_t \)
- \( n'_{t+1} \geq 0 \) and \( m'_{t+1} \geq 0 \)
An equilibrium process for prices \( \{p_t, q_t, w_t\} \) is such that

1. Entrepreneurs choose \( l_t \) to maximize \( y_t \) given \( k_t \), and choose \( \{c_t, i_t, k_{t+1}, n_{t+1}, m_{t+1}\} \) to maximize utility function subject to constraints.

2. Workers choose \( \{c'_t, l'_t, n'_{t+1}, m'_{t+1}\} \) to maximize utility function subject to constraints.

3. Markets for output, labor, equity and money clear.
Labor market equilibrium: $\left(\frac{w_t}{\omega}\right)^{1/\nu} = K_t \left[\left(1 - \gamma\right)\frac{A_t}{w_t}\right]^{1/\gamma}$, implying

- $r_t = a_t (K_t)^{\alpha - 1}$
- $a_t$ and $\alpha$ are derived from $A_t$, $\gamma$, $\omega$ and $\nu$, and $\alpha \in (0, 1)$

Behavior of entrepreneurs with an investing opportunity:

- $q_t < 1 \rightarrow$ not invest; rather buy equity
- $q_t = 1 \rightarrow$ indifferent
- $q_t > 1 \rightarrow$ invest by selling equity up to the constraint
Claim 1 If $\theta$ and $\phi$ satisfy $(1 - \lambda)\theta + \pi \lambda \phi > (1 - \lambda)(1 - \pi)$, then around the steady state $(A, \phi)$:

- (i) The allocation is first best
- (ii) $q_t = 1$
- (iii) $p_t = 0$
- (iv) $r_t \approx \frac{1}{\beta} - \lambda$

$\Rightarrow$ if liquidity constraint is loose enough, the equity market is sufficient to achieve the first best and money has no value.
Claim 2 If θ and φ are low enough, in the neighbourhood of the steady state:

(i) $p_t > 0$
(ii) $q_t > 1$
(iii) an entrepreneur with an investment opportunity spends entire money holdings: $m^i_{t+1} = 0$
In a monetary equilibrium, an investing entrepreneur’s constraints become:

\[
c_t^i + q_t^R n_{t+1} = r_t n_t + [\phi_t q_t + (1 - \phi_t)q_t^R] \lambda n_t + p_t m_t
\]

where \( q_t^R := \frac{1 - \theta q_t}{1 - \theta} < 1 \) is the effective replacement cost of equity.

Consumption and investment are given by:

\[
c_t^i = (1 - \beta) \left\{ r_t n_t + [\phi_t q_t + (1 - \phi_t)q_t^R] \lambda n_t + p_t m_t \right\}
\]

\[
i_t^i = \frac{(r_t + \lambda \phi_t q_t)n_t + p_t m_t - c_t^i}{1 - \theta q_t}
\]
Equilibrium Characterization

Saving entrepreneur’s consumption is given by

\[ c^s_t = (1 - \beta)(r_t n_t + q_t \lambda n_t + p_t m_t) \]

Remaining resources are split between \( m_{t+1} \) and \( n_{t+1} \), where the optimal split is determined by an Euler Equation:

\[
\begin{align*}
    u'(c_t) &= \mathbb{E}_t \left\{ \frac{p_{t+1}}{p_t} \left[ (1 - \pi)u'(c^s_{t+1}) + \pi u'(c^i_{t+1}) \right] \right\} \\
    &= (1 - \pi) \mathbb{E}_t \left\{ \frac{r_{t+1} + \lambda q_{t+1}}{q_t} u'(c^s_{t+1}) \right\} + \\
    &\quad \pi \mathbb{E}_t \left\{ \frac{r_{t+1} + \lambda \phi_{t+1} q_{t+1} + \lambda (1 - \phi_{t+1}) q^{R}_{t+1}}{q_t} u'(c^i_{t+1}) \right\}
\end{align*}
\]
Equilibrium Characterization

Linearity of decision rules facilitates aggregation. Aggregate investment is

\[(1 - \theta q_t)I_t = \pi \left\{ \beta \left[ (r_t + \lambda \phi_t q_t)K_t + p_t M \right] - (1 - \beta)(1 - \phi_t)q_t^R K_t \right\} \]

Goods market clearing requires output net of labor costs to equal consumption plus investment of entrepreneurs (workers consume their labor income):

\[a_t K_t^\alpha = l_t + \left\{ [r_t + (1 - \pi + \pi \phi)\lambda q_t + \pi(1 - \phi_t)\lambda q_t^R] + p_t M \right\} \]
Equilibrium Characterization

Aggregate counterpart of pricing equation:

\[(1 - \pi)E_t \left[ \frac{(r_{t+1} + \lambda q_{t+1})/q_t - p_{t+1}/p_t}{(r_{t+1} + q_{t+1}\lambda)N_{t+1}^s + p_{t+1}M} \right] \]

\[= \pi E_t \left[ \frac{p_{t+1}/p_t - [r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R]/q_t}{[r_{t+1} + \phi_{t+1}\lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R]N_{t+1}^s + p_{t+1}M} \right] \]

where \(N_{t+1}^s := \theta I_t + \phi_t \pi \lambda K_t + (1 - \pi)\lambda K_t\) is next period equity of savers.

- LHS: excess return of equity over money when there will not be an investment opportunity (liquidity premium of equity)
- RHS: larger return of money when there will be an investment opportunity (limited resaleability of equity)
Equilibrium Characterization

Steady State: 3 equations in $q$, $p$ and $K$.

Claim 3: In the neighbourhood of the steady state monetary economy,

(i) Capital stock is less than in a frictionless economy:

$$K_{t+1} < K^* \iff \mathbb{E}_t(a_{t+1}K_{t+1}^{\alpha-1} + \lambda) > \frac{1}{\beta}$$

(ii) \(\mathbb{E}_t\left(\frac{a_{t+1}K_{t+1}^{\alpha-1} + \lambda q_{t+1}}{q_t}\right) < \frac{1}{\beta}\)

(iii) \(\mathbb{E}_t\left(\frac{p_{t+1}}{p_t}\right) < \mathbb{E}_t\left(\frac{a_{t+1}K_{t+1}^{\alpha-1} + \lambda q_{t+1}}{q_t}\right)\)

(iv) \(\mathbb{E}_t\left(\frac{a_{t+1}K_{t+1}^{\alpha-1} + \phi_{t+1} + \lambda q_{t+1} + (1 - \phi_{t+1})\lambda q_{t+1}^R}{q_t}\right) < \mathbb{E}_t\left(\frac{p_{t+1}}{p_t}\right)\)
Markov process for $\phi_t$ with two states $\phi(1 + \Delta\phi)$, $\phi(1 - \Delta\phi)$ and constant probability of switch
Liquidity Shock under Laissez-Faire

\[ \phi_t \]

\[ \ln q_t \]

\[ \ln p_t \]

\[ \ln I_t \]

\[ \ln C_t \]
Dynamics

When low $\phi$ arrives:

- equity less valuable: $q_t$ falls and $p_t$ increases (flight to liquidity)
- investment decreases and remains at lower level
- government can offset effects of fluctuations in $\phi_t$ by printing money and purchasing equity, so investing entrepreneurs hold a larger amount of liquid money