Liquidity, Business Cycles and Monetary Policy

Nobuhiro Kiyotaki and John Moore

September 24, 2009

Introduction

Objective of the paper:

- Introduce a model with differences in liquidity across assets, where fiat money may have value in equilibrium
- Analyze the effects of aggregate productivity and liquidity shocks
- Study the role of open market operations where the government prints money to buy equity

Environment

4 objects traded:

- Nondurable general output
- Labor
- Equity
- Fiat money

2 types of agents:

- Entrepreneurs
- Workers

Environment: Entrepreneurs

- Preferences: $\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)$
- Access to production technology $y_t = A_t k_t^{\gamma} l_t^{1-\gamma}$
  $\rightarrow \mathrm{y}_t - w_t l_t = r_t k_t$
- Investment opportunity (to produce new capital) arrives with probability π, *iid* across agents and time
  - $k_{t+1} = \lambda k_t + i_t$

Environment: Entrepreneurs

Financing investment: equity issuance and (possibly) fiat money

2 key assumptions:

- Entrepreneur cannot pledge more than fraction θ of future returns from new capital [*borrowing constraint*]
- Entrepreneur can only sell up to fraction ϕ_t of equity [*resaleability constraint*]

Simplifying assumption: resaleability constraint affects inside equity (own unmortgaged capital) as well as outside equity

Environment: Entrepreneurs

n_t =total equity
m_t =fiat money holdings
s_t =equity issued

Constraints imply

- $n_{t+1} = \lambda n_t + i_t - s_t$
- $s_t \leq \theta i_t + \phi_t \lambda n_t$

Eliminate s_t:

- $n_{t+1} \geq (1 - \theta) i_t + (1 - \phi_t) \lambda n_t$; also
- $m_{t+1} \geq 0$
- $c_t + i_t + q_t(n_{t+1} - i_t - \lambda n_t) + p_t(m_{t+1} - m_t) = r_t n_t$

where q_t is the price of equity and p_t the price of fiat money

Environment: Workers

- Preferences: $\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_s' - \frac{\omega}{1+\nu} l_s'^{1+\nu})$

- Constraints:
  - $c_t' + q_t(n_{t+1}' - \lambda n_t') + p_t(m_{t+1}' - m_t') = w_t l_t' + r_t n_t'$
  - $n_{t+1}' \geq 0$ and $m_{t+1}' \geq 0$

Equilibrium

An equilibrium process for prices $\{p_t, q_t, w_t\}$ is such that

- (i) entrepreneurs choose l_t to maximize y_t given k_t, and choose $\{c_t, i_t, k_{t+1}, n_{t+1}, m_{t+1}\}$ to maximize utility function subject to constraints
- (ii) workers choose $\left\{c'_t, l'_t, n'_{t+1}, m'_{t+1}\right\}$ to maximize utility function subject to constraints
- (iii) markets for output, labor, equity and money clear

Equilibrium Characterization

Labor market equilibrium: $(w_t/\omega)^{1/\nu} = K_t\left[(1-\gamma)A_t/w_t\right]^{1/\gamma}$, implying

- $r_t = a_t(K_t)^{\alpha-1}$
- a_t and α are derived from A_t, γ, ω and ν, and $\alpha \in (0,1)$

Behavior of entrepreneurs with an investing opportunity:

- $q_t < 1 \rightarrow$ not invest; rather buy equity
- $q_t = 1 \rightarrow$ indifferent
- $q_t > 1 \rightarrow$ invest by selling equity up to the constraint

Equilibrium Characterization

Claim 1 If θ and ϕ satisfy $(1-\lambda)\theta + \pi\lambda\phi > (1-\lambda)(1-\pi)$, then around the steady state (A, ϕ)

- (i)The allocation is first best
- $(ii) q_t = 1$
- $(iii) p_t = 0$
- $(iv)\ r_t \approx \frac{1}{\beta} - \lambda$

$\Rightarrow$ if liquidity constraint is loose enough, the equity market is sufficient to achieve the first best and money has no value

Equilibrium Characterization

Claim 2 If θ and ϕ are low enough, in the neighbourhood of the steady state:

- (i) $p_t > 0$
- (ii) $q_t > 1$
- (iii) an entrepreneur with an investment opportunity spends entire money holdings: $m^i_{t+1} = 0$

Equilibrium Characterization

In a monetary equilibrium, an investing entrepreneur's constraints become:

$$c_t^i + q_t^R n_{t+1}^i = r_t n_t + [\phi_t q_t + (1-\phi_t) q_t^R] \lambda n_t + p_t m_t$$

where $q_t^R := \frac{1-\theta q_t}{1-\theta} < 1$ is the effective replacement cost of equity.

Consumption and investment are given by:

- $c_t^i = (1-\beta) \left\{ r_t n_t + \left[\phi_t q_t + (1-\phi_t) q_t^R \right] \lambda n_t + p_t m_t \right\}$
- $i_t^i = \frac{(r_t + \lambda \phi_t q_t) n_t + p_t m_t - c_t^i}{1-\theta q_t}$

Equilibrium Characterization

Saving entrepreneur's consumption is given by

$$c_t^s = (1-\beta)(r_t n_t + q_t \lambda n_t + p_t m_t)$$

Remaining resources are split between m_{t+1} and n_{t+1}, where the optimal split is determined by an Euler Equation:

$$\text{u'}(\text{c}_t) = \mathbb{E}_t \left\{ \frac{p_{t+1}}{p_t} \left[(1-\pi) u'(c_{t+1}^s) + \pi u'(c_{t+1}^i) \right] \right\}$$
$$= (1\text{-}\pi)\mathbb{E}_t \left\{ \frac{r_{t+1} + \lambda q_{t+1}}{q_t} u'(c_{t+1}^s) \right\} +$$
$$\pi \mathbb{E}_t \left\{ \frac{r_{t+1} + \lambda \phi_{t+1} q_{t+1} + \lambda(1-\phi_{t+1}) q_{t+1}^R}{q_t} u'(c_{t+1}^i) \right\}$$

Equilibrium Characterization

Linearity of decision rules facilitates aggregation. Aggregate investment is

$$(1-\theta q_t)I_t = \pi \left\{\beta \left[(r_t + \lambda\phi_t q_t)K_t + p_t M\right] - (1-\beta)(1-\phi_t)q_t^R K_t\right\}$$

Goods market clearing requires output net of labor costs to equal consumption plus investment of entrepreneurs (workers consume their labor income):

$$a_t K_t^\alpha = I_t + \left\{\left[r_t + (1-\pi+\pi\phi)\lambda q_t + \pi(1-\phi_t)\lambda q_t^R\right] + p_t M\right\}$$

Equilibrium Characterization

Aggregate counterpart of pricing equation:

$$(1-\pi)\mathbb{E}_t\left[\frac{(r_{t+1}+\lambda q_{t+1})/q_t - p_{t+1}/p_t}{(r_{t+1}+q_{t+1}\lambda)N^s_{t+1}+p_{t+1}M}\right]$$

$$= \pi\mathbb{E}_t\left[\frac{p_{t+1}/p_t-[r_{t+1}+\phi_{t+1}\lambda q_{t+1}+(1-\phi_{t+1})\lambda q^R_{t+1}]/q_t}{[r_{t+1}+\phi_{t+1}\lambda q_{t+1}+(1-\phi_{t+1})\lambda q^R_{t+1}]N^s_{t+1}+p_{t+1}M}\right]$$

where $N^s_{t+1} := \theta I_t + \phi_t \pi \lambda K_t + (1-\pi)\lambda K_t$ is next period equity of savers.

- LHS: excess return of equity over money when there will not be an investment opportunity (liquidity premium of equity)
- RHS: larger return of money when there will be an investment opportunity (limited resaleability of equity)

Equilibrium Characterization

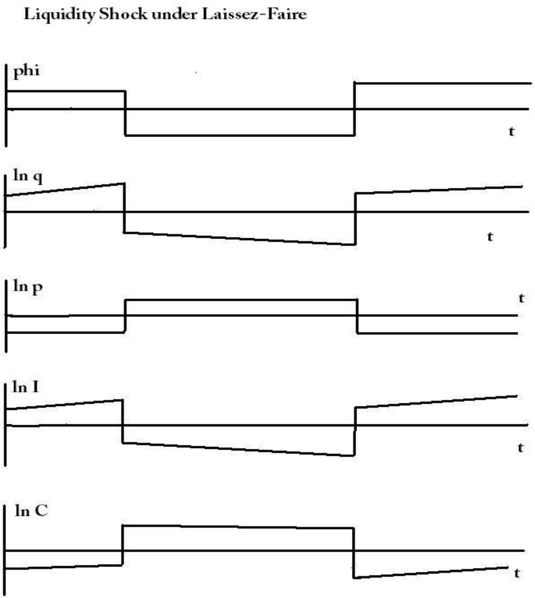
Steady State: 3 equations in q, p and K.
Claim 3: In the neighbourhood of the steady state monetary economy,

- *(i)* Capital stock is less than in a frictionless economy: $K_{t+1} < K^* \iff \mathbb{E}_t(a_{t+1}K_{t+1}^{\alpha-1} + \lambda) > \frac{1}{\beta}$
- *(ii)* $\mathbb{E}_t(\frac{a_{t+1}K_{t+1}^{\alpha-1}+\lambda q_{t+1}}{q_t}) < \frac{1}{\beta}$
- *(iii)* $\mathbb{E}_t(\frac{p_{t+1}}{p_t}) < \mathbb{E}_t(\frac{a_{t+1}K_{t+1}^{\alpha-1}+\lambda q_{t+1}}{q_t})$
- *(iv)* $\mathbb{E}_t(\frac{a_{t+1}K_{t+1}^{\alpha-1}+\phi_{t+1}\lambda q_{t+1}+(1-\phi_{t+1})\lambda q_{t+1}^R}{q_t}) < \mathbb{E}_t(\frac{p_{t+1}}{p_t})$

Dynamics

Markov process for ϕ_t with two states $\phi(1+\Delta_\phi)$, $\phi(1-\Delta_\phi)$ and constant probability of switch

Dynamics


Liquidity Shock under Laissez-Faire
phi
t
ln q
t
ln p
t
ln I
t
ln C
t

Dynamics

When low ϕ arrives:

- equity less valuable: q_t falls and p_t increases (flight to liquidity)
- investment decreases and remains at lower level
- government can offset effects of fluctuations in ϕ_t by printing money and purchasing equity, so investing entrepreneurs hold a larger amount of liquid money