Asset Pricing in a Production Economy with Chew-Dekel Preferences

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Recursive Utility

- The dimensions involved in a stochastic dynamic decision problem are essentially two: the time dimension and the risk dimension
- The function below defines the class of stationary recursive preferences

$$U(S_t) = W\left[c_t, \mu(S_t)\right] \tag{1}$$

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- The function below defines the class of stationary recursive preferences

$$U(S_t) = W\left[c_t, \mu(S_t)\right] \qquad (1)$$

- Constant elasticity time aggregator:

$$U(S_t) = \left[c_t^{\gamma} + \beta\mu(S_t)^{\gamma}\right]^{\frac{1}{\gamma}} \qquad \gamma \leq 1,\ \beta > 0 \qquad (2)$$

Generalized Disappointment Aversion (GDA)

$$\mu(S_t)^\eta = E_t[U(S_{t+1})^\eta] \overbrace{-\theta \sum_{S_{t+1} \in \triangle_{t+1}} \pi(S_{t+1} \mid S_t)[(\xi\mu(S_t))^\eta - U(S_{t+1})^\eta]}^{\text{penalty to disappointing events}}$$

$$\triangle_{t+1} = \{S_{t+1} : U(S_{t+1}) < \xi\mu(S_t)\} \qquad \eta \leq 1,\ \xi \geq 0,\ \theta \geq 0$$

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- $\{\theta = 0, \eta = \gamma\} \Rightarrow U(S_t) = \left[c_t^\gamma + \beta E_t[U(S_{t+1})^\gamma]\right]^{\frac{1}{\gamma}}$

GDA: Properties

- GDA preferences satisfy the weak independence axiom: *"independence only for lotteries that are disappointment-comparable"* (Gul, 1991)
- **First order risk aversion:** consider two states $w_0(1+k)$ and $w_0(1-k)$

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$$P(k) = \frac{\theta}{2+\theta}dk - \frac{1}{2}\frac{u''(w_0)w_0}{u'(w_0)}\left[\frac{4(1+\theta)}{(2+\theta)^2}\right]dk^2$$

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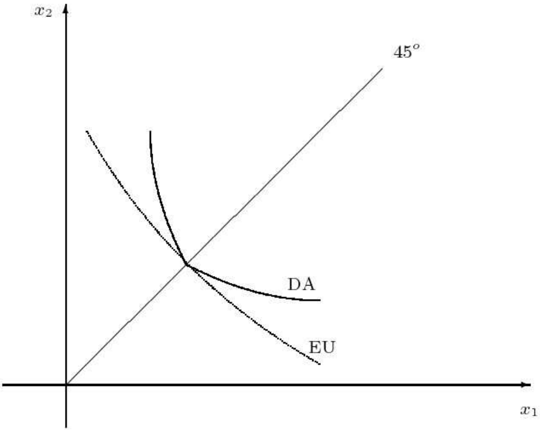
$\Rightarrow$ EZ preferences imply much higher risk aversion for higher risk (off the equilibrium path). This is not true for GDA preferences

EU vs DA



Risk aversion and risk

- simple atemporal bet: 1-κ and 1+κ with equal probability
- GDA preferences with DA parameter θ, $\xi = 1$ and $\eta = 1$
- EU preferences with RRA coefficient $1 - \eta$

$$\underbrace{\frac{1+\theta}{2+\theta}(1-\kappa) + \frac{1}{2+\theta}(1+\kappa)}_{\text{C.E. of DA agent}} = \underbrace{\left[\frac{1}{2}(1-\kappa)^{\eta} + \frac{1}{2}(1+\kappa)^{\eta}\right]^{\frac{1}{\eta}}}_{\text{C.E. of EU agent}}$$

- the larger the risk, the lower the $1 - \eta$ required to generate the same C.E. of a given θ

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	$\kappa = 0.002$	$\kappa = 0.2$
DA ($\theta = .04$)	1.00	0.99
EU ($\eta = -17.53$)	1.00	0.83

Risk aversion and risk

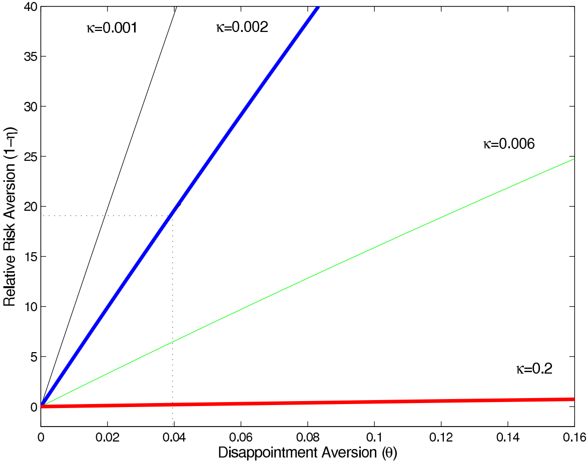

Figure 4: Comparison between risk–preferences.

This paper

- Real business cycle model that tests the assets pricing properties of 4 different production economies
  - Expected discounted utility
  - Epstein-Zin utility
  - DA preferences
  - GDA preferences

Technology

$$
\begin{aligned}
y_t &= k_t^{\alpha}(z_t l_t)^{1-\alpha} \\
z_t &= e^{\lambda t+\varepsilon_t} \qquad \lambda > 0 \\
\varepsilon_t &= \rho\varepsilon_{t-1} + \zeta_t \qquad \zeta_t \sim N(0,\sigma^2)
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k_{t+1} &= i_t + (1-\delta)k_t \\
y_t &= c_t + i_t + g(k_t, k_{t+1}) \\
g(k_t, k_{t+1}) &= \left| \frac{k_{t+1}}{k_t} - \psi \right|^{\iota} k_t \qquad \iota > 1, \psi > 0
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Inelastic labor supply: $l = 1$, variables in per-capita terms

Planner's Problem

$$
\begin{aligned}
v(k,\varepsilon) &= \max_{k'} \left\{ c^{\gamma} + \beta e^{[\varphi+\gamma\lambda]} \mu[v(k',\varepsilon')]^{\gamma} \right\}^{\frac{1}{\gamma}} \\
s.t. & \\
c + e^{(\lambda+\varphi)} k' &= k^{\alpha}(e^{\varepsilon} l)^{1-\alpha} + (1-\delta)k - \left| e^{(\lambda+\varphi)} \frac{k'}{k} - \psi \right|^{\iota} k \\
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\varepsilon' &= \rho\varepsilon + \zeta' \qquad \zeta' \sim N(0,\sigma^2) \\
\mu^{\eta} &= E_{\varepsilon'} \left\{ \frac{[1+\theta I(k',\varepsilon',\mu)] v^{\eta}(k',\varepsilon')}{1+\theta E_{\varepsilon'}[I(k',\varepsilon',\mu)]} \right\} \\
I(k',\varepsilon',\mu) &= 1 \qquad \textit{if } v(k',\varepsilon') < \xi\mu \text{ , } \textit{0 otherwise}
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Assumption $\psi = e^{\lambda+\varphi} \Rightarrow$ balanced growth path invariant to ι

Calibration: Epstein-Zin

α	$=$	0.36	
δ	$=$	0.025	
λ	$=$	0.00387	annual g.r. per-capita output
φ	$=$	0.00298	annual g.r. of population
β	$=$	1.147	using restrictions of balanced growth
γ	$=$	-38.86	relative quarterly std of cons. growth
η	$=$	-17.53	quarterly risk-free rate
θ	$=$	0	
ξ	$=$	1	
ρ	$=$	0.95	
σ	$=$	0.0164	std of output growth
ι	$=$	1.2153	quarterly equity premium

Calibration: DA

α	=	0.36	
δ	=	0.025	
λ	=	0.00387	annual g.r. per-capita output
φ	=	0.00298	annual g.r. of population
β	=	1.147	using restrictions of balanced growth
γ	=	-38.86	relative quarterly std of cons. growth
η	=	1	quarterly risk-free rate
θ	=	0.133*	
ξ	=	1	
ρ	=	0.95	
σ	=	0.0164	std of output growth
ι	=	1.2153	quarterly equity premium

∗ unconditional risk-free rate, unconditional equity premium, relative std of consumption growth

Calibration: GDA

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λ	=	0.00387	annual g.r. per-capita output
φ	=	0.00298	annual g.r. of population
β	=	1.147	using restrictions of balanced growth
γ	=	-38.86	relative quarterly std of cons. growth
η	=	1	quarterly risk-free rate
θ	=	1.8	
ξ	=	1.0069	
ρ	=	0.95	
σ	=	0.0164	std of output growth
ι	=	1.2153	quarterly equity premium

Unconditional moments

	σ_C/σ_Y	σ_C/σ_Y	$E(r^f)$	Std(r^f)	$E(r^e - r^f)$	Std(r^e)
E-Z	0.501	2.516	0.250%	2.96%	1.889%	12.16%
DA	0.492	2.525	0.255%	3.03%	1.836%	12.487%
GDA	0.499	2.395	0.241%	2.662%	1.855%	12.431%
Data	0.499	2.647	0.252%	0.834%	1.893%	7.694%

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Other test: predictability of equity premium far from the one implied by the data

Comments

- An economy with DA agents produces results very similar to a standard EZ agents economy
- DA agents show *more reasonable* risk aversion in environment riskier than the one implied by the model
- comparative statics could give different results. Assume a drop in the volatility of TFP then the impact would be larger under EZ (second order risk aversion) than under DA (first order risk aversion)

Comments

- A comparative static exercise (i.e. drop in volatility of TFP) should be done
- **High volatility of risk-free rate**