A Liquidity-Based Model of Security Design

DeMarzo and Duffie
Econometrica 1999

Sargent's Reading Group
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  - cash flow always greater than some value $\rightarrow$ issue risk-free debt equal to this value and retain all remaining cash flow: no illiquidity cost but high retention cost;
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- Optimal securities: not necessarily standard debt. Example: tranches of CDOs partly retained, partly sold.

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$$\sup_{q \in [0,1]} \delta E(X - F|Z) + \delta(1-q)E(F|Z) + qP_F(q) =$$

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- **Definition:** A Bayes-Nash equilibrium for the liquidation game is a pair (P, Q):
  - $Q(\tilde{f}) \in \arg\max_q q \left(p - \delta \tilde{f}\right)$ almost surely;
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- Implied **profit** of public offering:

$$\Pi_F(f) = \Pi(f, f_0) = (1-\delta)f^{1/(1-\delta)}f_0^{-\delta/(1-\delta)}$$

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- **Proposition:** Suppose Z can be contractible. Then there exists an optimal security F whose private valuation does not depend on Z, in particular satisfying $\tilde{f} = f_0 = \min_z E_z(X)$. This security is not retained by the issuer, that is $Q^*(\tilde{f}) = 1$. Ex: if $Z = X$, issue a security $F = \min(X)$.

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- **Numerical example:**
  - $X = (1 + Z)Y$, $Y = \exp\left[\alpha - \sigma^2/2 + \sigma e\right]$, $e \sim N(0, 1)$.
  - $Z \sim U(-m, m)$ with $m \in (0, 1)$; here $z_0 = -m$ is a uniform worst case: thus, standard debt $F = \min(X, d)$ is optimal.
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  - Securitization profit for different levels of d and m.
- **Proposition:** adding a risk-free cash flow to X increases debt capacity d by less than one for one. Scope for multiple tranches: risk free (senior), risky (mezzanine), ...

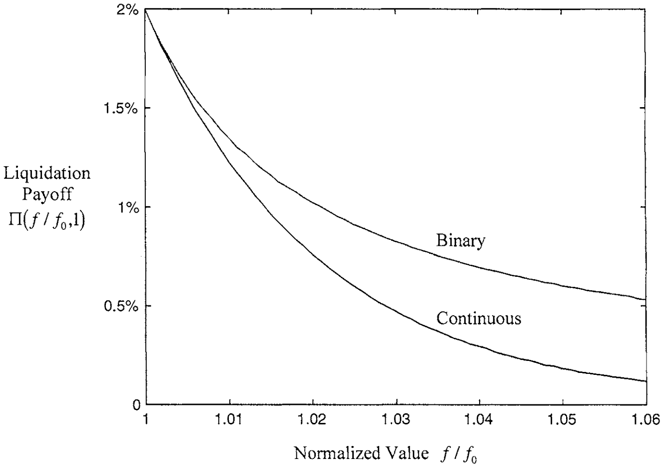

FIGURE 3.—Profit of public offering.

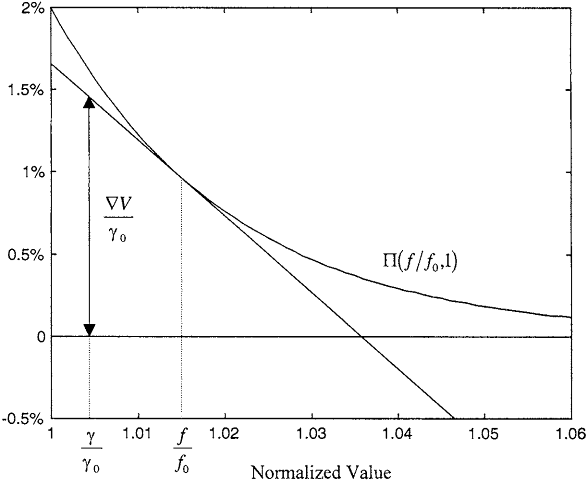

FIGURE 4.—Marginal gain of adding G to F.

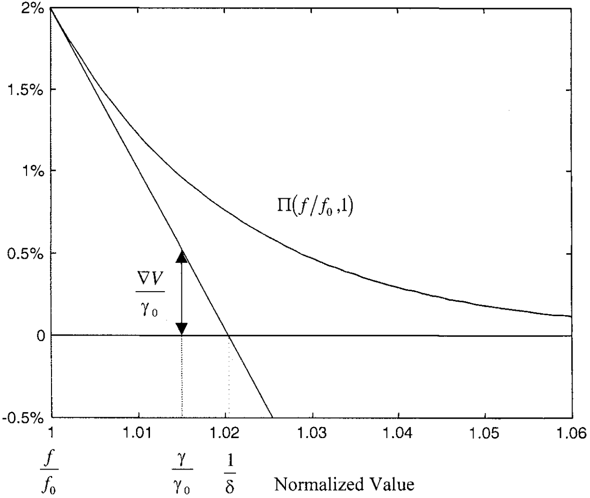

FIGURE 5.—Adding risky cash flows to a risk-free security.

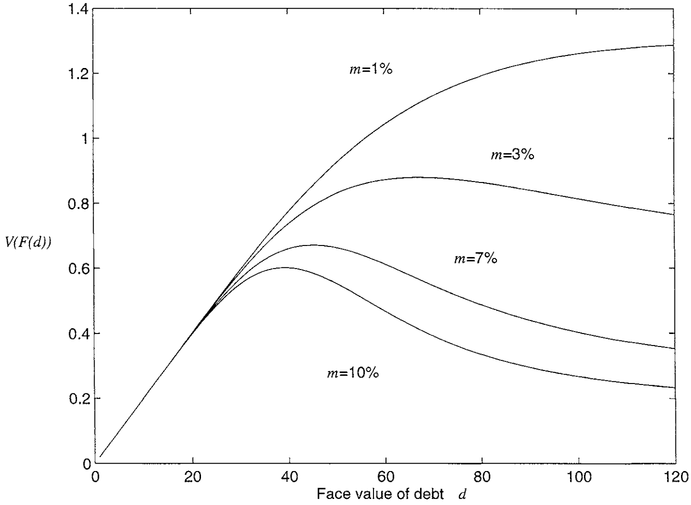

FIGURE 6.—Securitization profit $V(F(d))$ at debt level d.