Bequests, Gifts, and Social Security

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Goal of Paper

- Povides a theoretical model illustrating possible effects of an unfunded social security system
- Novel feature: Transfers can go from parents to children and from children to parents
- Primary finding: Possibility of two-sided transfers may make social security less harmful to private wealth accumulation
- Secondary finding: Risk-loving behavior is likely to arise for households with low levels of wealth

The Set-Up

- OLG: parent and child
- Each household lives two periods:
  - Parent and child overlap for one period
  - Working period: family raises one child
  - Retirement: child forms own household and passes first period of life
- Labor supply: inelastic in first period
- Labor earnings: $\tilde{A} \in \mathbb{A} \equiv \left[A^L, A^U\right]$ determines wA_t
- Savings: actuarially fair lotteries
- Social security system introduced later

Preferences

- A family born at time t ranks allocations according to:

$$h \underbrace{v(c_{2,t-1})}_{\text{parent}} + \underbrace{u(c_{1,t})}_{\text{child period 1}} + E\left\{ \underbrace{v(c_{2,t})}_{\text{child period 2}} + \sum_{i=1}^{\infty} h^i \underbrace{[u(c_{1,t+i}) + v(c_{2,t+i})]}_{\text{decendants}} \right\}, \tag{1}$$

- $h \in (0,1)$
- $v(c) \equiv au(c), 1 > a > 0$
- Expectation over labor earnings and lottery outcomes
- Wealth of child at time t:

$$W_t = wA_t + \underbrace{B_t - G_t}_{\text{net bequest}}$$

- Consumption of parent at time t:

$$G_t + L(S_{t-1}) = B_t + c_{2,t-1}$$

Decisions & Timing

- First: Child's earnings and the outcome of the parent's lottery are realized
- Second: Parent and child simultaneously choose transfers within the sets:

$$0 \leq B_t \leq L(S_{t-1}), \quad 0 \leq G_t \leq wA_t.$$

- Third: Parent derives $C_{2,t-1}$ and child chooses S_t and C_{1t} subject to:

$$W_t = S_t + C_{1t}, \quad S_t \geq 0$$

- Life-cycle savings are invested in an actuarially fair lottery $\tilde{L}(S_t)$ i.e.

$$r \cdot S_t = \sum_{i=1}^{m} q_i l_i$$

- Let $\mathbb{L}$ be the set of all such lotteries $(l_1, q_1; \ldots; l_m, q_m)$, $l_i \geq 0$

Solution Concept

Stationary subgame perfect Nash equilibrium

- Define the value of having wealth W after transfers have taken place:

$$V(W; w) \equiv u(c_{1,t}) + E\left\{ v(c_{2,t}) + \sum_{i=1}^{\infty} h^{i} \left[u(c_{1,t+i}) + v(c_{2,t+i}) \right] \right\}$$

- Furthermore, define (1) for the descendant as:

$$\delta(L, N; w; A; V) \equiv hv(L - N) + V(wA + N; w)$$

and for the parent as:

$$\Delta(L, N; w; A; V) \equiv v(L - N) + hV(wA + N; w).$$

$\Rightarrow$ refer to before the game is played

- For given lottery outcome L, given earning wA_{t+1}:

$$B = \arg \sup_{B' \in [0, L+G]} \Delta(L, B' - G; w; A_{t+1}; V_{t+1}) \tag{2}$$

$$G = \arg \sup_{G' \in [0, wA_{t+1}+B]} \delta(L, B - G'; w; A_{t+1}; V_{t+1}) \tag{3}$$

- Then the net bequest $N = B - G$ depends on:

$$N = N(L; w; A_{t+1}, V_{t+1})$$

- For a given lottery outcome L, integrate utility of parent over ability of child:

$$\Gamma(L; w; V) \equiv \int_{\mathbb{A}} \Delta(L, N(L(S); w; A_{t+1}, V_{t+1}); w; \tilde{A}; V) p(A) dA$$

- We can write the value function as:

$$V(W; w) = \sup_{S \in [0, W], \tilde{L}(S) \in \mathbb{L}} \left\{ u(W - S) + E\left[\Gamma(\tilde{L}(S); w; V) \right] \right\}$$

Value Function

If V is a solution it must be strictly concave

- Define:

$$\psi(w;V) \equiv conv\left(\{(L,\Gamma(L;w;V))|L \geq 0\}\right)$$

- and:

$$\Psi(L;w;V) \equiv \sup\left\{Z : (L,Z) \in \psi(w;V)\right\}$$

which is concave in L.

- Note that $\Psi(\cdot;w;V) \geq \Gamma(\cdot;w;V)$ (draw graph)
- The interpretation of $conv(\cdot)$ and the availability of lotteries yield:

$$\begin{aligned} V(W;w) &= \sup_{S\in[0,W]\tilde{L}(S)\in\mathbb{L}} \left\{u(W-S) + E\left[\Gamma(\tilde{L}(S);w;V)\right]\right\} = \\ &= \sup_{S\in[0,W]} \left\{u(W-S) + \Psi(rS;w;V)\right\} \end{aligned}$$

Net Bequest Function

- Allow parent complete control over its net transfer:

$$N_{\Delta}(L; w; A; V) \equiv \arg \sup_{N \in [-wA, L]} \Delta(L, N; w; A; V)$$

- Allowed descendant's family complete control over its net transfer:

$$N_{\delta}(L; w; A; V) \equiv \arg \sup_{N \in [-wA, L]} \delta(L, N; w; A; V)$$

- Imperfect altruism implies that $N_{\delta} \geq N_{\Delta}$
- If $N_{\Delta} > 0$ actual net bequest will be positive
- If $N_{\delta} < 0$ actual net bequest will be negative
- If $N_{\Delta} < 0$ and $N_{\delta} > 0$, the parent would like a gift and the offspring would like a bequest but cannot compel it; hence we observe $N = 0$ (draw graph)

Lotteries

Convexities are likely to arise for families with low wealth

- Define:

$$Y(L; w; A; V) \equiv \Delta(L; N(L; w; A; V); w; A; V)$$

- Take derivative from the left:

$$\frac{\partial^-}{\partial L}\left[v(L_0 - N(L_0; w; A; V)) + hV(wA + N(L_0; w; A; V); w)\right] =$$

$$= v'(c_2) - \left[v'(c_2) - hV_W\right] \frac{\partial^- N}{\partial L}$$

- Take derivative form the right: $\frac{\partial^+ Y}{\partial L} = v'(c_2)$
- Convexity at L_0 (draw graph):

$$\frac{\partial^- Y}{\partial L} < \frac{\partial^+ Y}{\partial L}$$

- Intuition: to the right \$1 more in wealth can be consumed; to the left \$1 more in wealth but net bequest is reduced

Wealth Accumulation

- For given r and w can find $V(\cdot)$ which in turn yields unique $S(\cdot)$ and $N(\cdot)$
- Use $S(\cdot)$ and $N(\cdot)$ to obtain a stationary distribution of normalized wealth
- Normalize W_t dividing through by w $X_t = W_t/w$:

$$X_{t+1} = A_{t+1} + N(L(S(X_t; 1; V)); 1; A_{t+1}, V) \tag{4}$$

  (Note: linear homogeneity properties have been established)
- Equation (4) induces a Markov transition function for X
- Cross-sectional distribution of normalized wealth converges to stationary distribution $D(\cdot)$ i.e. $\tilde{X} \sim D$
- Mean normalized savings per family:

$$\frac{K_t}{wN_t} = E\left[S(\tilde{X}; 1; V)\right]$$

- For differing values of r we can plot the supply curve

Social Security

- Want to compare steady-state supply curves with (S^*) and without (S) social security
- Finance pay-as-you-go social security with lump sum tax on young families: $w \cdot \tau$
- Benefit in old-age: $\gamma \cdot w \cdot \tau$ where γ is the gross rate of labor-augmenting technological progress
- The present value of each household's tax less benefit is σw, where:

$$\sigma \equiv \left[\frac{r-\gamma}{r}\right]\tau \quad \text{i.e.} \quad w \cdot \tau - \frac{\gamma \cdot w \cdot \tau}{r} = w \cdot \sigma$$

- At a first level of analysis ...
- Net lifetime labor earnings changes from $w \cdot A$ to $w \cdot (A - \sigma)$
- Compute stationary distribution of normalized wealth $\tilde{X}(\sigma)$
- One might imagine that with an unfunded social security system, the supply of wealth is given by:

$$E\left[S(\tilde{X}(\sigma); 1; V)\right] - \frac{\tau\gamma}{r} \tag{5}$$

- Each household's need for life-cycle savings is reduced by the present value of future social security benefits

- At a second level of analysis ...
- Solution $V(\cdot)$ and associated $S(\cdot)$ and $N(\cdot)$ will change after $t = 0$
- Only discusses what intuitively may happen
- Equation (5) presupposes that families which have savings of $S = S_0 < (\tau\gamma/r)$ in the absence of social security would borrow $S_0 - (\tau\gamma/r)$
- Due to the no-borrowing constraints, the savings of such families cannot be reduced and the horizontal shift from S to S^* will be cut back

- In the pre-social security world low income families may choose low savings in order to extract a larger gift
- Social security may prevent the family from extracting a gift from its descendant
- Furthermore, marginal value of saving may increase
- By being forced into the concave region of Γ may increase demand for life insurance and annuities