Identification with Taylor Rules: A Critical Review

John Cochrane

- Can monetary policy rules a la Taylor be identified from data?
- simple example:

$$i_t = r + \phi\pi_t + x_t \tag{1}$$

- in particular, can we determine whether or not monetary policy was 'active' or 'passive'?

Identification

Let B be a set of observables (e.g. an infinite time series on inflation) and θ the set of parameters of the model. This paper deals with 'General Identification', i.e. the existence of a one-to-one mapping between B and θ

A small example

$$i_t = r + \phi \pi_t + x_t \tag{2}$$

$$x_t = \rho x_{t-1} + \varepsilon_t \tag{3}$$

$$i_t = r + E_t \pi_{t+1} \tag{4}$$

Substituting out the nominal interest rate gives:

$$E_t \pi_{t+1} = \phi \pi_t + x_t \tag{5}$$

This can be solved forward (assuming $\phi > 1$):

$$\pi_t = -\sum_{j=0}^{\infty} \frac{1}{\phi^{j+1}} E_t x_{t+j} = \frac{x_t}{\rho - \phi} \tag{6}$$

Dynamics of inflation:

$$\pi_t = \rho\pi_{t-1} - \varepsilon_t/(\phi - \rho) \tag{7}$$

Plug this into the Fisher relation:

$$i_t = r + \rho\pi_t \tag{8}$$

Note that IV estimation does not help here (if we don't assume further equations).
One can find a model with $\phi < 1$ that has the same reduced form dynamics.

A prototypical NK model

$$i_t = \phi_{\pi,0}\pi_t + \phi_{\pi,1}E_t\pi_{t+1} + \phi_{y,0}y_t + \phi_{y,1}E_t y_{t+1} + x_{it} + \theta_y x_{yt} + \theta_\pi x_{\pi t} \quad (9)$$

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t\pi_{t+1}) + x_{yt} \quad (10)$$

$$\pi_t = \beta E_t\pi_{t+1} + \gamma y_t + x_{\pi t} \quad (11)$$

$$x_{it} = \rho_i x_{it-1} + \varepsilon_{it} \quad (12)$$

$$x_{yt} = \rho_y x_{yt-1} + \varepsilon_{yt} \quad (13)$$

$$x_{\pi t} = \rho_\pi x_{\pi t-1} + \varepsilon_{\pi t} \quad (14)$$

VAR representation of observables

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} = B \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} + C \begin{bmatrix} \varepsilon_{yt+1} \\ \varepsilon_{\pi t+1} \\ \varepsilon_{it+1} \end{bmatrix} \tag{15}$$

Assumption: the ε's can be arbitrarily correlated (so C is not helping with identification).

There are 9 parameters in B but 12 parameters in the model, so we already know that we can't identify all parameters.

This can be rewritten as:

$$\begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} = Q\varrho Q^{-1} \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} + Q\varepsilon_{zt+1} \tag{16}$$

$$\varrho = \begin{bmatrix} \rho_i & 0 & 0 \\ 0 & \rho_y & 0 \\ 0 & 0 & \rho_\pi \end{bmatrix} \tag{17}$$

Results

1. ϱ and β, γ, σ are identified
2. knowledge of ϱ and β, γ, σ does not help to identify the policy rule parameters
3. if ϕ is known, one can identify θ
4. if $\theta = 0$ any two of the ϕ parameters can be identified
5. all 4 elements of ϕ can not be separately identified
6. if both θ elements are allowed to be different from zero, no element of ϕ can be identified

Lubik and Schorfheide

consider again the simple model, this time with

$$x_t = \varepsilon_t \tag{18}$$

Generically we have

$$\pi_{t+1} = \phi\pi_t + \varepsilon_t + \delta_{t+1} \tag{19}$$

where δ_{t+1} is a forecast error. If $\phi > 1$

$$\pi_t = -\frac{\varepsilon_t}{\phi} \tag{20}$$

If $\phi < 1$ (and assuming $\delta_t = M\varepsilon_t$)

$$\pi_t = \phi\pi_{t-1} + \varepsilon_{t-1} + M\varepsilon_t \tag{21}$$

- This means that indeterminacy can be tested by using a likelihood ratio test for an ARMA(1,1) model vs. white noise.
- Note that the assumption of $x_t = \varepsilon_t$ is crucial.
- π_t can generally be an ARMA process in the determinacy region.