The conquest of US inflation: Learning and robustness to model uncertainty

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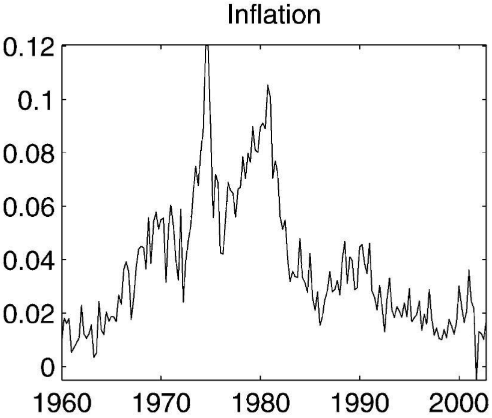
A look at US inflation



How to explain this pattern

Key ingredients in the literature seem to be:

- A drifting natural rate of unemployment
- the central bank has to learn about this and about the (Phillips curve) tradeoffs it faces (e.g. Primiceri (2006) and Sargent, Williams and Zha (forthcoming))
- *But it is pretty hard to come up with a story that explains why the Fed did not disinflate much earlier*

The story of this paper

The authors attribute the slow reaction of the Fed to model uncertainty (in addition to learning): what kind of Phillips curve should we believe in?

- Solow-Samuelson:

$$y_t = \gamma_0 + \gamma_1(L)y_{t-1} + \gamma_2(L)u_t + \eta_{1t} \tag{1}$$

- Solow-Tobin:

$$\Delta y_t = \delta_1(L)\Delta y_{t-1} + \delta_2(L)(u_t - u_t^*) + \eta_{2t} \tag{2}$$

- Lucas-Sargent:

$$u_t - u_t^* = \phi_1(y_t - x_{t|t-1}) + \phi_2(L)(u_{t-1} - u_{t-1}^*) + \eta_{3t} \tag{3}$$

policy instrument:

$$y_t = x_{t|t-1} + \xi_t \tag{4}$$

The Fed's decision problem

1. Given data up until $t-1$, update estimates of each Phillips curve separately using a normal - inverse gamma prior and observations of u^*_{t-1} generated by:

$$u^*_t = u^*_{t-1} + 0.075(u_t - u^*_{t-1}) \tag{5}$$

2. given these estimates, update model probabilities (because of the normal - inverse gamma setup, a closed form solution is available)
3. solve a LQ optimal regulator problem as follows (but implement only x_t) :

$$\min_{x_t} L_E = \alpha_{1t} L(M_1) + \alpha_{2t} L(M_2) + \alpha_{3t} L(M_3) \tag{6}$$

where $L(M_i)$ is given by

$$L(M_i) = E_{t,i} \sum_{j=0}^{\infty} \beta^j (u^2_{t+j} + \lambda y^2_{t+j}) \tag{7}$$

The LQ problem in more detail

- The loss functions can be stacked into a single (probability weighted) loss function, so we can apply standard LQ algebra (see RMT)
- You might wonder what the right constraint for the LQ is. Let S_{Et} be the stacked vector of states necessary for all 3 models. Then the sequence of constraints is given by:

$$S_{Et+j} = A_E(t-1)S_{Et+j-1} + B_E(t-1)x_{t+j|t-1} + C_E(t-1)\eta_t \quad (8)$$

- optimal policy:

$$x_{t|t-1} = -f_E(t-1)S_{Et-1} \quad (9)$$

Bounded rationality

The decision maker assumes that the estimates of each Phillips curve and the model probabilities are known with certainty when he makes his decision (i.e. parameter uncertainty is not taken into account). This is a common approach in the macroeconomic learning literature ("Anticipated Utility") and rules out experimentation to improve parameter estimates, which is something that seems to be very much in line with actual central bank behavior (Blinder, Lucas)

A robust decision rule

- If the composite model is detectable and stabilizable:
  - standard algorithms can be used to compute its solution
  - the eigenvalues of $A_E - B_E f_E$ are less than $\beta^{-1/2}$ in absolute value
  - the expected loss is finite, which (with positive probability weights on each model) also implies that each model's loss is finite
- On the other hand, an infinite loss under one model implies an infinite loss in the composite model. This means that the regulator must avoid an infinite loss under any model with positive probability, no matter how small that probability may be, leading to a robust decision rule
- *This turns out to be the reason behind the Fed's hesitation to disinflate according to this paper!*

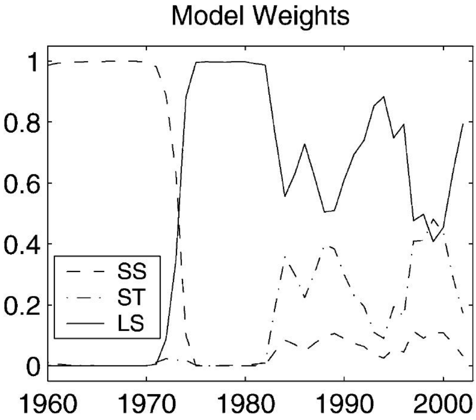
Some details

The estimation is started in 1960. Priors for each Phillips curve come from regressions based on a training sample, prior model probabilities are set to 0.98 for Samuelson-Solow and 0.01 for each of the other two models. β and λ are fixed at 0.99 and 14

Table 1
Lag order in central bank approximating models

	Inflation	Unemployment
Samuelson–Solow	$\gamma_1 : 4$	$\gamma_2 : 2$
Solow–Tobin	$\delta_1 : 3$	$\delta_2 : 2$
Lucas–Sargent	$\phi_1 : 0$	$\phi_2 : 2$

Results



Results II

Note that under the timing assumptions of the paper, $x_t = 0$ would be optimal under the Lucas-Sargent model. But why didn't the Fed implement that policy if it strongly believed in the model?

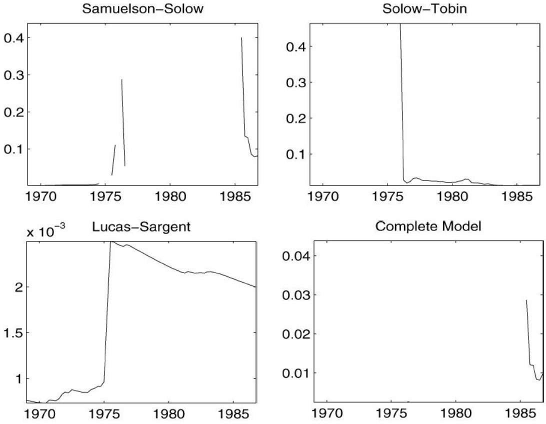


Fig. 3. Expected loss of a zero inflation policy.

Eigenvalues

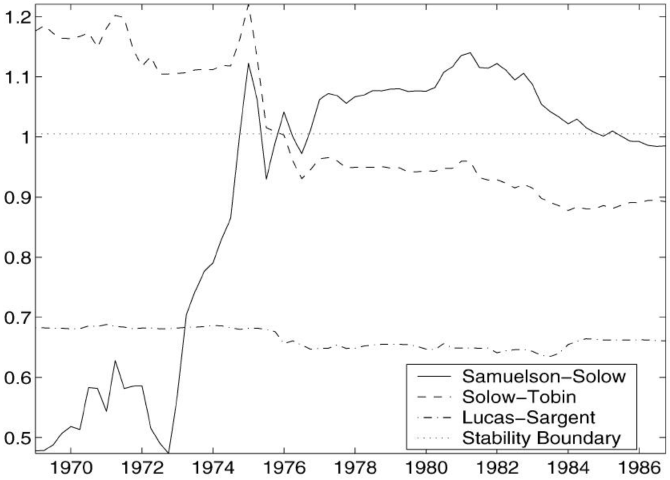


Fig. 4. Dominant eigenvalue under zero inflation.

Optimal Policy

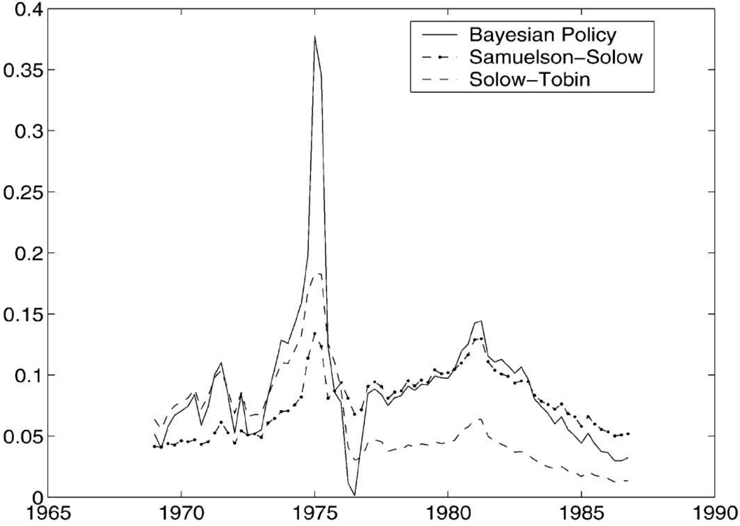


Fig. 7. Optimal policy and policy for worst-case scenarios.

Optimal Policy II

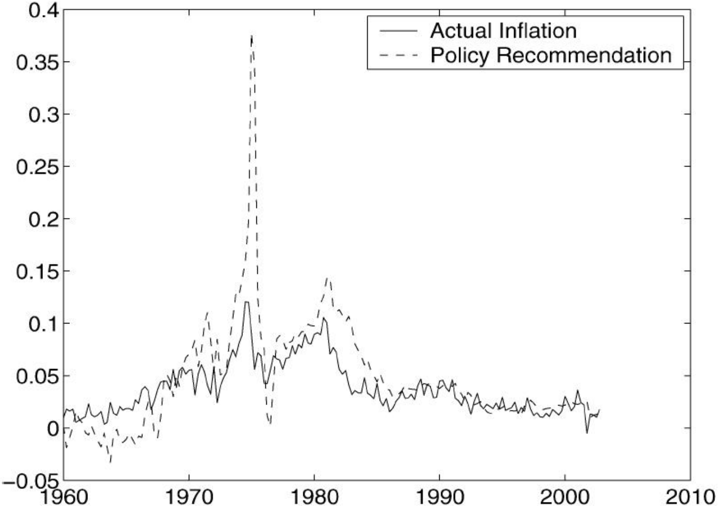


Fig. 2. Inflation and optimal policy.

The Phillips curve: direction of fit matters

Table 2
Sacrifice ratios and the direction of fit

	SS-K	*ST-K*	*SS-C*	*ST-C*	LS
1970.Q4	0.295	0.607	0.036	0.014	0
1975.Q4	0.578	0.249	−0.009	0.006	0
1979.Q4	0.636	0.227	−0.008	0.007	0

Note: Percent output loss associated with a 1 percentage point reduction in inflation sustained for 8 quarters.

Two brief comments

- It would be interesting to see how a Taylor rule would do compared to the optimal Bayesian policy (one would have to augment the model with a relationship between interest rates and the economy, say a Fisher relation)
- One could (at a maybe unbearable computational cost) introduce another loss function to study issues such as the zero bound on nominal interest rates