Time-Varying Risk, Interest Rates and Exchange Rates in General Equilibrium

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The Risk Premium p_t

$$p_t = i_t^* + E_t \log e_{t+1} - e_t - i_t \tag{1}$$

- if exchange rates follow a random walk then all differences in interest rates are driven by the risk premium
- $$cov(i_t - i_t^*, \log e_{t+1} - e_t) \leq 0 \tag{2}$$
- 'Fama Regression':

$$\log e_{t+1} - e_t = a + b(i_t - i_t^*) + u_{t+1} \tag{3}$$

Environment

- two country economy, 'foreign' variables are denoted with a $*$
- Households are indexed by γ (density $f(\gamma)$)
- at the beginning of period 1 households of type γ have M_0 units of home currency, $\overline{B}_h(\gamma)$ units of the home government debt and $\overline{B}_h^*(\gamma)$ units of the foreign government debt
- M_t is the stock of money and μ_t its growth rate from t-1 to t

Market Structure

- aggregate event $s_t = (\mu_t, \mu_t^*)$ with density over histories $g(s^t)$
- state contingent nominal bonds issued by the governments are traded in each country
- imposing no arbitrage, we can assume that agents in each country only trade in their countries bonds.

Households

- $$\sum_{t=1}^{\infty} \beta^t \int U(c(s^t, \gamma)) g(s^t) ds^t \tag{4}$$

- $$c(s^t, \gamma) = n(s^t, \gamma) + x(s^t, \gamma) z(s^t, \gamma) \tag{5}$$

- $$n(s^{t+1}, \gamma) = \frac{P(s^t) y}{P(s^{t+1})} \tag{6}$$

- $$B(s^t, \gamma) = \int_{s_{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}, \gamma) + P(s^t)[x(s^t, \gamma) + \gamma] z(s^t, \gamma) \tag{7}$$

- $$\int [c(s^t,\gamma) + \gamma z(s^t,\gamma)] f(\gamma) d\gamma = y \tag{8}$$

- $$\int [n(s^t,\gamma) + (\gamma + x(s^t,\gamma)) z(s^t,\gamma))] f(\gamma) d\gamma = M(s^t)/P(s^t) \tag{9}$$

- $$\int_{s_{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}) ds_{t+1} + M(s^t) - M(s^{t-1}) = B(s^t) \tag{10}$$

Planning Problem

- $$\sum_{t=1}^{\infty} \beta^t \int_{\gamma} \int_{s^t} U(c(s^t,\gamma)) f(\gamma) g(s^t) d\gamma ds^t \tag{11}$$
- subject to feasibility and
$$c(s^t,\gamma) = z(s^t,\gamma) c_A(s^t,\gamma) + [1 - z(s^t,\gamma)] y/\mu_t \tag{12}$$
- FOC for c_A:
$$\beta^t U'(c_A(s^t,\gamma)) g(s^t) = \lambda(s^t) \tag{13}$$

Planning Problem II

- equilibrium consumption is given by:

$$c(s^t,\gamma) = y/\mu_t \forall\gamma \leq \bar{\gamma}(\mu_t) \tag{14}$$

$$c(s^t,\gamma) = c_A(\mu_t) \forall\gamma > \bar{\gamma}(\mu_t) \tag{15}$$

- where $c_A(\mu)$ and $\bar{\gamma}(\mu)$ are the solutions to:

$$U(c_A(\mu))F(\bar{\gamma}(\mu)) + U(y/\mu)[1 - F(\bar{\gamma}(\mu))] \tag{16}$$

subject to

$$c_A(\mu)F(\bar{\gamma}(\mu)) + \int_0^{\bar{\gamma}(\mu)} \gamma f(\gamma)d\gamma + y/\mu[1 - F(\bar{\gamma}(\mu))] = y \tag{17}$$

Stochastic Discount Factors

- $$m(s^t, s_{t+1}) = \beta \frac{U'(c_A(\mu_{t+1}))}{U'(c_A(\mu_t))} \frac{1}{\mu_{t+1}} \tag{18}$$

- $$i_t = -\log E_t m_{t+1} \tag{19}$$

- $$1 = E_t m_{t+1} R_{t+1} = E_t[m_{t+1} \frac{e_{t+1}}{e_t} R^*_{t+1}] \tag{20}$$

- $$\log e_{t+1} - \log e_t = \log m^*_{t+1} - \log m_{t+1} \tag{21}$$

- $$p_t = E_t \log m^*_{t+1} - E_t \log m_{t+1} - E_t(\log m^*_{t+1} - \log m_{t+1}) \tag{22}$$

Segmentation and Inflation

Theorem

As μ increases, more households become active. In particular, $\bar{\gamma}'(\mu) > 0$ for $\mu > 1$ and $\bar{\gamma}'(1) = 0$

Theorem

$\log(c_A)$ *is strictly increasing and strictly concave in* $\log \mu$ *around* $\mu = 1$

Approximate Pricing Kernel

- $$\log U'(c_A(\mu_t)) = \log U'(c_A(\bar{\mu})) - \phi\widehat{\mu}_t + .5\eta\widehat{\mu}_t^2 \tag{23}$$
- $$\log m_{t+1} = \log\beta - \log(\bar{\mu}) - (\phi+1)\widehat{\mu}_{t+1} + .5\eta\widehat{\mu}_{t+1}^2 + \phi\widehat{\mu}_t - 5\eta\widehat{\mu}_t^2 \tag{24}$$

Linking Money Growth and Risk Premia

- $$\widehat{\mu}_t = E_{t-1}\widehat{\mu}_t + \varepsilon_t \tag{25}$$

- $$p_t = 0.5\frac{1}{1-\eta\sigma_\varepsilon^2}(var_t \log m_{t+1} - var_t \log m^*_{t+1}) \tag{26}$$

- $$var_t \log m_{t+1} = [-(1+\phi) + \eta E_t\widehat{\mu}_{t+1}]^2\sigma_\varepsilon^2 + \frac{3}{4}\eta^2\sigma_\varepsilon^4 \tag{27}$$

- $$\frac{dp_t}{d\widehat{\mu}_t} = -\frac{\eta(\phi+1)}{1-\eta\sigma_\varepsilon^2}\frac{dE_t\widehat{\mu}_{t+1}}{d\widehat{\mu}_{t+1}} \tag{28}$$

Exchange Rates

Theorem

if

$$1 - \eta\sigma_\varepsilon^2 < \frac{1+\phi}{\phi}\frac{dE_t\widehat{\mu}_{t+1}}{d\widehat{\mu}_{t+1}} \leq 1 \tag{29}$$

then for μ_t close to $\bar{\mu}$ a change in money growth raises $i_t - i_t^*$ and lowers $E_t \log e_{t+1} - \log e_t$

Long Term Risk Premia

$$p_{t,k} = E_t[\frac{1}{k}\log\frac{e_{t+k}}{e_t}] - (i_{t,k} - i^*_{t,k}) \tag{30}$$

Theorem

Suppose

$$\lim_{k\to\infty}\frac{1}{k}\log E_t[\frac{P_t}{P_{t+k}}] = \lim_{k\to\infty}\frac{1}{k}\log E_t[\exp(-\sum_{s=1}^{k}\log\mu_{t+s})] < \infty \tag{31}$$

and

$$c_A(\mu) \geq \underline{c} > 0\forall\mu \tag{32}$$

Then the long term risk premium in this model is the same as that in a model with $\gamma = 0$ *for all agents*