In Search of a Theory of Debt Management

Authors: Faraglia, Marcet, Scott

Presentation: Dan Greenwald

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Complete Market Approach to Debt Management

- Lucas and Stokey (1983) solve the Ramsey problem with complete markets, describes optimal fiscal policy for government.
- However, depends crucially on government using state-contingent debt: Aiyagari, Marcet, Sargent, and Seppälä (2002) show that optimal policy is sharply different when government is restricted to one-period non-contingent bonds.
- Angeletos (2002) shows that if the government has access to enough bonds of different maturities, then generically a portfolio can be formed that implements the optimal policy.
- This literature also offers qualitative policy implications: issue long-term debt and hold short-term claims.
- In contrast, this paper will show that using the maturity structure to implement the complete markets allocation is likely to require extreme and risky policies, and does not offer clear qualitative policy prescriptions.

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- Technology:

$$c_t + g_t \leq \theta_t(1 - x_t)$$

where c_t, g_t, θ_t, x_t are consumption, government spending, productivity, and leisure. Define $s_t = (g_t, \theta_t)'$.

- Preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + v(x_t)\right].$$

- Consumer's budget constraint:

$$z_0 = \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) \left[c_t(s^t) - (1 - \tau_t^x(s^t))w_t(s^t)(1 - x_t(s^t))\right]$$

where z_0 is initial consumer assets (government debt).

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- Substitute consumer's FOCs to obtain implementability constraint:

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- Applying state prices from consumer's FOC yields government debt at node s^t:

$$z_t = \frac{1}{u'(c_t)} \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[u'(c_{t+j}) c_{t+j} - v'(x_{t+j})(1 - x_{t+j}) \right]$$

- Under the complete markets solution, allocations (c_t, x_t) at time t depend only on the current state, s_t.

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Completing the Market via the Maturity Structure

- Assume that there are N possible realizations of the shocks, so that $s_t \in \{\bar{s}_1, \ldots, \bar{s}_N\}, \forall t$, and the government is restricted to trading non-contingent bonds of N maturities $b_t^1, \ldots, b_t^N$, with prices $p_t^1, \ldots, p_t^N$.
- Government's budget constraint

$$g_t + \sum_{j=1}^{N} p_t^{j-1} b_{t-1}^j \leq \tau_t^x w_t (1 - x_t) + \sum_{j=1}^{N} p_t^j b_t^j$$

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$$p_t^j = \beta^t \frac{\mathbb{E}_t u'(c_{t+j})}{u'(c_t)}.$$

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$$\sum_{j=1}^{N} p_t^{j-1}(s_t, s^{t-1}) b_{t-1}^j(s^{t-1}) = z_t(s_t, s^{t-1}), \quad \forall s_t$$

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- Formally, this means solving the problem

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  which yields a unique solution if bond prices are linearly independent across states.

- If this holds, the government can implement the complete markets outcome using only non-contingent bonds.

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Simple Example

- Consider the case where $N = 2$, $\theta_t = \bar{\theta}$, and $g_t \in \{\bar{g}_L, \bar{g}_H\}$ follows the Markov chain
$$\begin{bmatrix} \pi_{HH} & 1 - \pi_{HH} \\ 1 - \pi_{LL} & \pi_{LL} \end{bmatrix}$$
- Assume that $\bar{g}_H > 0$, $\bar{g}_L < 0$, $z_0 = 0$, $g_0 = \bar{g}_H$, so that $\bar{z}_H = 0$, $\bar{z}_L > 0$.
- Bond portfolio is given by
$$\begin{bmatrix} b^1_{t-1} \\ b^2_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & \bar{p}_H \\ 1 & \bar{p}_L \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \bar{z}_L \end{bmatrix} = \begin{bmatrix} \bar{p}_H \bar{z}_L / (\bar{p}_H - \bar{p}_L) \\ -\bar{z}_L / (\bar{p}_H - \bar{p}_L) \end{bmatrix}$$
- Since $\bar{p}_H < \bar{p}_L$, optimal policy is to issue long-term debt and hold short-term claims.
- Intuition: long-term debt has a low return in state $\bar{g}_H$, reducing debt when government spending is high.
- If the one-period-ahead variability of long rates $\bar{p}_H - \bar{p}_L$ is not large (as in calibrations), then very large positions are required to attain the complete markets allocation.

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Numerical Example

- Simulation results using CRRA utility (bond positions as fractions of output).

Table 1
Simulation results—endowment economy.[a]

Shocks						**Interest rates**				
g							H	L		
		B^1	B^{30}			R^1	2.23	1.85		
	$\mu=1$	-7.04	7.16			R^{30}	2.10	1.98		
		B^1	B^{30}			R^1	3.95	0.13		
	$\mu=0$	-0.79	0.81			R^{30}	2.28	1.80		
$\boldsymbol{\theta}$							H	L		
		B^1	B^{30}			R^1	1.07	2.93		
	$\mu=1$	-0.85	0.90			R^{30}	1.85	2.21		
		B^1	B^{30}			R^1	-3.13	7.21		
	$\mu=0$	-0.17	0.18			R^{30}	1.86	2.21		
g,$\boldsymbol{\theta}$							HH	HL	LH	LL
		B^1	B^4	B^{13}	B^{30}	R^1	1.23	3.25	0.90	2.71
	$\mu=1$	-16.15	41.32	-86.71	57.66	R^{30}	1.92	2.28	1.79	2.15
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-5.75	7.21	-2.98	4.16
$\pi^\theta_{HH}=0.91$	$\mu=1/3$	-4.22	58.48	-161.22	106.37	R_{29}	1.92	2.28	1.79	2.14
		B^1	B^5	B^{18}	B^{30}	R^1	2.00	2.45	1.64	2.71
	$\mu=1$	63.82	-140.94	163.15	-75.64	R^{30}	1.97	2.22	1.85	2.09
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-3.34	6.96	-2.74	4.02
$\pi^\theta_{HH}=0.98$	$\mu=1/3$	5.77	-85.8	210.19	-129.51	R^{29}	1.91	2.28	1.79	2.14

[a] Table shows maturity structure and yield curve for endowment economy subject to various combinations of productivity and expenditure shocks.

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- Notation: μ is fraction of calibrated persistence.

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$\pi^\theta_{HH}=0.91$	$\mu=1/3$	-4.22	58.48	-161.22	106.37	R_{29}	1.92	2.28	1.79	2.14
		B^1	B^5	B^{18}	B^{30}	R^1	2.00	2.45	1.64	2.71
	$\mu=1$	63.82	-140.94	163.15	-75.64	R^{30}	1.97	2.22	1.85	2.09
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-3.34	6.96	-2.74	4.02
$\pi^\theta_{HH}=0.98$	$\mu=1/3$	5.77	-85.8	210.19	-129.51	R^{29}	1.91	2.28	1.79	2.14

[a] Table shows maturity structure and yield curve for endowment economy subject to various combinations of productivity and expenditure shocks.

Numerical Example

- Positions are huge: 4 to 160 times GDP at each maturity.

Table 1
Simulation results—endowment economy.[a]

Shocks						**Interest rates**				
g							H	L		
		B^1	B^{30}			R^1	2.23	1.85		
	$\mu=1$	-7.04	7.16			R^{30}	2.10	1.98		
		B^1	B^{30}			R^1	3.95	0.13		
	$\mu=0$	-0.79	0.81			R^{30}	2.28	1.80		
$\boldsymbol{\theta}$							H	L		
		B^1	B^{30}			R^1	1.07	2.93		
	$\mu=1$	-0.85	0.90			R^{30}	1.85	2.21		
		B^1	B^{30}			R^1	-3.13	7.21		
	$\mu=0$	-0.17	0.18			R^{30}	1.86	2.21		
$\mathbf{g},\boldsymbol{\theta}$							HH	HL	LH	LL
		B^1	B^4	B^{13}	B^{30}	R^1	1.23	3.25	0.90	2.71
	$\mu=1$	-16.15	41.32	-86.71	57.66	R^{30}	1.92	2.28	1.79	2.15
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-5.75	7.21	-2.98	4.16
$\pi^\theta_{HH}=0.91$	$\mu=1/3$	-4.22	58.48	-161.22	106.37	R_{29}	1.92	2.28	1.79	2.14
		B^1	B^5	B^{18}	B^{30}	R^1	2.00	2.45	1.64	2.71
	$\mu=1$	63.82	-140.94	163.15	-75.64	R^{30}	1.97	2.22	1.85	2.09
$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-3.34	6.96	-2.74	4.02
$\pi^\theta_{HH}=0.98$	$\mu=1/3$	5.77	-85.8	210.19	-129.51	R^{29}	1.91	2.28	1.79	2.14

[a] Table shows maturity structure and yield curve for endowment economy subject to various combinations of productivity and expenditure shocks.

Numerical Example

- Optimal portfolio varies dramatically with small changes in maturity.

Table 1
Simulation results—endowment economy.[a]

Shocks						**Interest rates**				
g							H	L		
		B^1	B^{30}			R^1	2.23	1.85		
	$\mu = 1$	−7.04	7.16			R^{30}	2.10	1.98		
		B^1	B^{30}			R^1	3.95	0.13		
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	$\mu = 1$	−0.85	0.90			R^{30}	1.85	2.21		
		B^1	B^{30}			R^1	−3.13	7.21		
	$\mu = 0$	−0.17	0.18			R^{30}	1.86	2.21		
$\mathbf{g}, \boldsymbol{\theta}$							HH	HL	LH	LL
		B^1	B^4	B^{13}	B^{30}	R^1	1.23	3.25	0.90	2.71
	$\mu = 1$	−16.15	41.32	−86.71	57.66	R^{30}	1.92	2.28	1.79	2.15
$\pi^g_{HH} = 0.95$		B^1	B^2	B^3	B^{29}	R^1	−5.75	7.21	−2.98	4.16
$\pi^\theta_{HH} = 0.91$	$\mu = 1/3$	−4.22	58.48	−161.22	106.37	R_{29}	1.92	2.28	1.79	2.14
		B^1	B^5	B^{18}	B^{30}	R^1	2.00	2.45	1.64	2.71
	$\mu = 1$	63.82	−140.94	163.15	−75.64	R^{30}	1.97	2.22	1.85	2.09
$\pi^g_{HH} = 0.95$		B^1	B^2	B^3	B^{29}	R^1	−3.34	6.96	−2.74	4.02
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[a] Table shows maturity structure and yield curve for endowment economy subject to various combinations of productivity and expenditure shocks.

Numerical Example

- In the two-shock model, changing the persistence of shocks can flip the signs of the positions.

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Simulation results—endowment economy.[a]

Shocks						**Interest rates**				
g							H	L		
		B^1	B^{30}			R^1	2.23	1.85		
	$\mu=1$	-7.04	7.16			R^{30}	2.10	1.98		
		B^1	B^{30}			R^1	3.95	0.13		
	$\mu=0$	-0.79	0.81			R^{30}	2.28	1.80		
$\boldsymbol{\theta}$							H	L		
		B^1	B^{30}			R^1	1.07	2.93		
	$\mu=1$	-0.85	0.90			R^{30}	1.85	2.21		
		B^1	B^{30}			R^1	-3.13	7.21		
	$\mu=0$	-0.17	0.18			R^{30}	1.86	2.21		
g,$\boldsymbol{\theta}$							HH	HL	LH	LL
		B^1	B^4	B^{13}	B^{30}	R^1	1.23	3.25	0.90	2.71
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$\pi^g_{HH}=0.95$		B^1	B^2	B^3	B^{29}	R^1	-5.75	7.21	-2.98	4.16
$\pi^\theta_{HH}=0.91$	$\mu=1/3$	-4.22	58.48	-161.22	106.37	R_{29}	1.92	2.28	1.79	2.14
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[a] Table shows maturity structure and yield curve for endowment economy subject to various combinations of productivity and expenditure shocks.

Complete Markets with Capital

- Technology:

$$c_t + g_t + k_t - (1-\delta)k_{t-1} \leq \theta_t k_{t-1}(1-x_t)^{1-\alpha}.$$

- Consumer's budget constraint

$$z_0 + k_{-1} = \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) \Big\{ c_t(s^t) + k_t(s^t) - R_t^k(s^t) k_{t-1}(s^{t-1}) - (1 - \tau_t^x(s^t)) w_t(s^t)(1 - x_t(s^t)) \Big\}$$

$$R_t^k(s^t) = \left[(1 - \tau_t^k(s^{t-1})) r_t(s^t) + (1-\delta) \right]$$

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$$R_t^k(s^t) = \big[(1 - \tau_t^k(s^{t-1})) r_t(s^t) + (1-\delta)\big]$$

Complete Markets with Capital

- Additional constraint for Ramsey planner:

$$u'(c_t) = \beta \mathbb{E}_t \left[u'(c_{t+1}) R^k_{t+1} \right]$$

- Optimal debt level

$$z_t = \frac{1}{u'(c_t)} \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[u'(c_{t+j}) c_{t+j} - v'(x_{t+j})(1 - x_{t+j}) \right] \right\} - R^k_t k_{t-1}$$

- Chari et al (1994) show that the solution satisfies the recursive structure

$$(k_t, c_t, x_t, \tau^x_t, \tau^k_{t+1})' = G(s_t, k_{t-1})$$

- Marcet and Scott (2009): this implies the existence of a time-invariant function D such that

$$D(k_{t-1}, s_t) = z^k_t(s^{t-1}, s_t)$$

so the optimal debt position depends only on the current state and the capital stock.

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so the optimal debt position depends only on the current state and the capital stock.

Completing the Market via the Maturity Structure

- To complete the market with maturities, we now need to solve

$$\begin{bmatrix} 1 & p_t^1(k_{t-1},\bar{s}_1) & \cdots & p_t^{N-1}(k_{t-1},\bar{s}_1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & p_t^1(k_{t-1},\bar{s}_N) & \cdots & p_t^{N-1}(k_{t-1},\bar{s}_N) \end{bmatrix} \begin{bmatrix} b_{t-1}^1(k_{t-1}) \\ \vdots \\ b_{t-1}^N(k_{t-1}) \end{bmatrix} = \begin{bmatrix} D(k_{t-1},\bar{s}_1) \\ \vdots \\ D(k_{t-1},\bar{s}_N) \end{bmatrix}$$

Numerical Results With Capital

- Notation: $E_{\pm 5\%}$ are the 5% lowest and highest positions for each maturity.

Table 2
Simulation results—capital accumulation.[a]

Shocks							Interest rates			
g		B^1	B^{30}				H			L
	$\mu=1$	−14.49	12.36			R^1	2.08			1.98
	$E_{+5\%}$	−18.29	9.41			R^{30}	2.07			2.00
	$E_{-5\%}$	−11.65	16.3							
		B^1	B^{30}							
	$\mu=0$	−9.23	7.19			R^1	2.06			1.99
	$E_{+5\%}$	−9.50	6.90			R^{30}	2.04			2.03
	$E_{-5\%}$	−8.94	7.46							
θ		B^1	B^{30}				H			L
	$\mu=1$	−8.49	6.26			R^1	2.26			1.85
	$E_{+5\%}$	−12.5	3.56			R^{30}	2.01			2.07
	$E_{-5\%}$	−5.62	10.10							
		B^1	B^{30}							
	$\mu=0$	−3.49	1.47			R^1	2.01			2.07
	$E_{+5\%}$	−3.93	1.19			R^{30}	2.02			2.06
	$E_{-5\%}$	−3.12	1.82							
g,**θ**							HH	HL	LH	LL
		B^1	B^4	B^{16}	B^{30}					
	$\mu=1$	−30.10	42.54	−48.18	33.44	R^1	2.46	1.67	2.26	1.48
	$E_{+5\%}$	−34.33	26.14	−97.58	15.94	R^{30}	2.03	2.07	1.94	1.98
	$E_{-5\%}$	−26.30	63.28	−16.46	66.29					
$\pi_H^g=0.95$		B^1	B^9	B^{13}	B^{29}					
$\pi_H^\theta=0.91$	$\mu=1/3$	−14.38	32.62	−30.74	10.42	R^1	2.04	1.97	2.00	1.92
	$E_{+5\%}$	−18.80	26.24	−36.75	8.16	R^{29}	2.01	2.05	2.00	2.03
	$E_{-5\%}$	−11.00	41.44	−25.37	11.91					
		B^1	B^5	B^{18}	B^{30}					
	$\mu=1$	−77.85	153.10	−207.77	130.19	R^1	2.55	1.63	2.42	1.50
	$E_{+5\%}$	−109.15	138.74	−226.37	106.12	R^{30}	2.09	2.05	2.02	1.99
	$E_{-5\%}$	−55.63	167.34	−189.63	161.17					
$\pi_H^g=0.95$		B^1	B^9	B^{14}	B^{29}					
$\pi_H^\theta=0.98$	$\mu=1/3$	−12.58	21.44	−23.13	12.20	R^1	2.07	1.94	2.03	1.90
	$E_{+5\%}$	−34.93	13.46	−54.90	8.63	R^{29}	2.03	2.00	2.05	2.01
	$E_{-5\%}$	−5.48	70.24	−18.56	17.44					

[a] Table shows maturity structure and yield curve for simulations of an economy with capital subject to productivity and expenditure shocks.

Numerical Results With Capital

- Adding capital introduces time variation of debt, and generally makes positions even larger.

Table 2
Simulation results—capital accumulation.[a]

Shocks							Interest rates				
g				B^1		B^{30}			H		L
		$\mu=1$		−14.49		12.36	R^1		2.08		1.98
		$E_{+5\%}$		−18.29		9.41	R^{30}		2.07		2.00
		$E_{-5\%}$		−11.65		16.3					
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$\boldsymbol{\theta}$				B^1		B^{30}			H		L
		$\mu=1$		−8.49		6.26	R^1		2.26		1.85
		$E_{+5\%}$		−12.5		3.56	R^{30}		2.01		2.07
		$E_{-5\%}$		−5.62		10.10					
				B^1		B^{30}					
		$\mu=0$		−3.49		1.47	R^1		2.01		2.07
		$E_{+5\%}$		−3.93		1.19	R^{30}		2.02		2.06
		$E_{-5\%}$		−3.12		1.82					
g,$\boldsymbol{\theta}$								HH	HL	LH	LL
			B^1	B^4	B^{16}	B^{30}					
		$\mu=1$	−30.10	42.54	−48.18	33.44	R^1	2.46	1.67	2.26	1.48
		$E_{+5\%}$	−34.33	26.14	−97.58	15.94	R^{30}	2.03	2.07	1.94	1.98
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		$E_{+5\%}$	−18.80	26.24	−36.75	8.16	R^{29}	2.01	2.05	2.00	2.03
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		$E_{+5\%}$	−34.93	13.46	−54.90	8.63	R^{29}	2.03	2.00	2.05	2.01
		$E_{-5\%}$	−5.48	70.24	−18.56	17.44					

[a] Table shows maturity structure and yield curve for simulations of an economy with capital subject to productivity and expenditure shocks.

Numerical Results With Capital

- Intuition: capital gives agents another instrument to smooth consumption, so bond prices are less volatile.

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g			B^1	B^{30}				H			L
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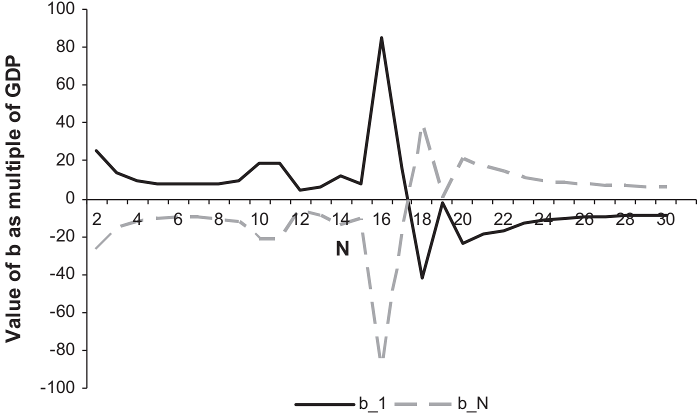
		$E_{-5\%}$	−5.62	10.10							
			B^1	B^{30}							
		$\mu=0$	−3.49	1.47			R^1	2.01			2.07
		$E_{+5\%}$	−3.93	1.19			R^{30}	2.02			2.06
		$E_{-5\%}$	−3.12	1.82							
g,θ								HH	HL	LH	LL
			B^1	B^4	B^{16}	B^{30}					
		$\mu=1$	−30.10	42.54	−48.18	33.44	R^1	2.46	1.67	2.26	1.48
		$E_{+5\%}$	−34.33	26.14	−97.58	15.94	R^{30}	2.03	2.07	1.94	1.98
		$E_{-5\%}$	−26.30	63.28	−16.46	66.29					
$\pi_H^g=0.95$			B^1	B^9	B^{13}	B^{29}					
$\pi_H^\theta=0.91$		$\mu=1/3$	−14.38	32.62	−30.74	10.42	R^1	2.04	1.97	2.00	1.92
		$E_{+5\%}$	−18.80	26.24	−36.75	8.16	R^{29}	2.01	2.05	2.00	2.03
		$E_{-5\%}$	−11.00	41.44	−25.37	11.91					
			B^1	B^5	B^{18}	B^{30}					
		$\mu=1$	−77.85	153.10	−207.77	130.19	R^1	2.55	1.63	2.42	1.50
		$E_{+5\%}$	−109.15	138.74	−226.37	106.12	R^{30}	2.09	2.05	2.02	1.99
		$E_{-5\%}$	−55.63	167.34	−189.63	161.17					
$\pi_H^g=0.95$			B^1	B^9	B^{14}	B^{29}					
$\pi_H^\theta=0.98$		$\mu=1/3$	−12.58	21.44	−23.13	12.20	R^1	2.07	1.94	2.03	1.90
		$E_{+5\%}$	−34.93	13.46	−54.90	8.63	R^{29}	2.03	2.00	2.05	2.01
		$E_{-5\%}$	−5.48	70.24	−18.56	17.44					

[a] Table shows maturity structure and yield curve for simulations of an economy with capital subject to productivity and expenditure shocks.

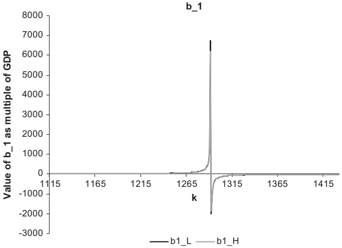
Numerical Results With Capital

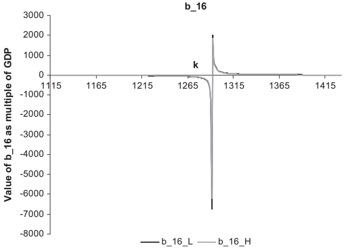
- In simple example with g_t fixed, two states for θ_t, and two maturities $(1, N)$, positions are volatile in N, and flip sign as N increases.



Numerical Results With Capital

- With N fixed at 16, the positions are discontinuous in capital, with an asymptote.





Habit Preferences

- Positions are so large in previous examples in part because there is unrealistically little variation in bond prices.

Table 3
Simulation results—consumption habits.[a]

Habits						Interest rates				
θ **shock**										
							H	L		
$\chi = 0$		B^1	B10			R^1	1.07	2.93		
	$\mu = 1$	-1.03	1.07			R^{10}	1.58	2.47		
$\chi = 0.273$		B^1	B^{10}							
	$\mu = 1$	-0.63	0.62			R^1	-0.58	5.10		
	$E_{+5\%}$	-0.68	0.59			R^{10}	1.37	2.73		
	$E_{-5\%}$	-0.58	0.66							
g,θ **shocks with** $\pi^g_{HH} = 0.95$ **and** $\pi^\theta_{HH} = 0.91$										
							HH	HL	LH	LL
$\chi = 0$		B^1	B^{10}	B^{16}	B^{30}	R^1	1.23	3.15	0.90	2.71
	$\mu = 1$	-4.60	71.74	-159.02	101.39	R^{30}	1.92	2.28	1.79	2.15
$\chi = 0.25$		B^1	B10	B15	B22					
	$\mu = 1$	-0.48	-18.23	7.01	11.48	R^1	0.09	5.39	-0.77	3.50
	$E_{+5\%}$	-0.50	-27.45	-91.14	-62.69	R^{22}	1.82	2.44	1.62	2.21
	$E_{-5\%}$	-0.45	-7.24	90.36	99.10					

Habit Preferences

- To address this, add a habit term to consumption, so preferences are over $u(c_t - \chi c_{t-1})$, and calibrate to match volatility of slope of yield curve.

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Habit Preferences

- Adding habit reduces the size of positions, but makes them extremely volatile.

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Robustness Concerns

- Previous results show that required positions are large and volatile (and therefore highly counterfactual), but in principle should complete the market.
- However, the extreme nature of the positions may create severe problems if the government's technology is not perfect.
- Even tiny levels of transactions costs (0.003% of the value of the positions) causes a balanced budget to deliver higher welfare than the complete markets allocation.
- Even without transactions costs, if the government is using a misspecified or miscalibrated model, the incorrect complete markets policy may be worse than a balanced budget policy.
- For experiments, welfare is presented using the statistic

$$R = \frac{W_X - W_{BB}}{W_{CM} - W_{BB}}$$

where (W_X, W_{BB}, W_{CM}) represents welfare under the experiment, balanced budget, and complete markets, respectively.

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Experiment 1: Misperception of Persistence Parameters

- Assume simple model with no capital, a two-state Markov chain for g_t, and that the government can hold bonds of maturities 1 and N.

Table 4
Robustness misperceptions across maturities.[a]

π^*_{HH}	max$R$	**R**	min$R$
0.99	−1.384	−3.319	−8.290
0.98	0.377	−0.067	−2.740
0.97	0.841	0.750	−0.026
0.96	0.974	0.962	0.818
0.95	1.000	1.000	1.000
0.94	0.987	0.982	0.727
0.93	0.959	0.944	−0.434
0.92	0.927	0.901	−3.269
0.91	0.894	0.857	−8.840
0.90	0.863	0.815	−17.998
0.89	0.834	0.776	−30.565
0.88	0.807	0.739	−44.899
0.87	0.782	0.705	−58.576
0.86	0.760	0.674	−69.745
0.85	0.739	0.645	−77.849
0.84	0.720	0.618	−83.321
0.83	0.703	0.593	−86.930
0.82	0.687	0.570	−89.365
0.81	0.672	0.548	−91.104

[a] Welfare loss across maturities when government misspecifies persistence of shocks in an endowment model with expenditure shocks ($R(\cdot)=1$ no welfare loss, $R(\cdot)<0$ balanced budget dominates).

Experiment 1: Misperception of Persistence Parameters

- Notation: R is the welfare statistic for $N = 30$. $\max R$ is under the best $N \in [2, 30]$. $\min R$ is under the worst $N \in [2, 30]$.

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Experiment 1: Misperception of Persistence Parameters

- Substantial welfare losses relative to CM even in best case, worst cases catastrophically worse than BB.

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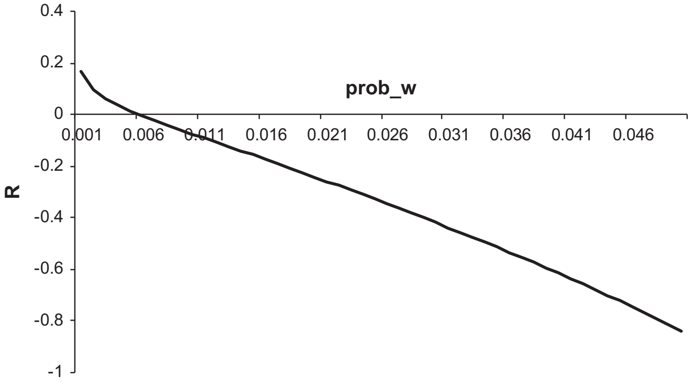
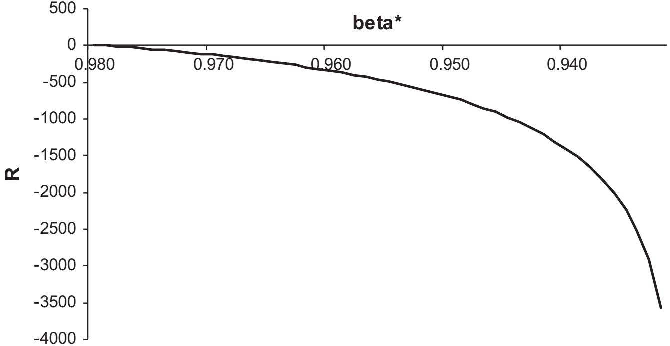
Experiment 2: Unperceived Disaster State

- Now assume that the government believes that g_t follows a two-value Markov chain, but in fact there is a third "disaster" state with very high expenditures.



Experiment 3: Misperception of Discount Factor

- You get the idea...



Conclusion

- Results show that completing markets using the maturity structure requires huge (and counterfactual) asset positions.
- Even qualitative implications are difficult to pin down — sign of positions is very sensitive to model, parameters, and state variables.
- Extreme positions needed to complete markets may lead to welfare losses relative to balanced budget if the government's technology is imperfect.
- Aside: could a government with robust concerns or transactions costs design a policy that is better than balanced budget?

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