Time-Consistent Debt

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An optimal fiscal policy problem:

- Lucas-Stokey (1983)
- Without commitment - Markov Perfect Equilibria

Three questions:

- What is time-inconsistent in the Ramsey plan?
- How do policies change, without commitment?
- How far are the commitment and the non-commitment plans?
Environment

- Time is discrete: $t = 0, 1, 2, ...$
- A representative household:
  - Values consumption and dislikes labor $U(c, n)$
  - Savings $b$ (complete markets)
- A representative firm:
  - Produces output with a linear technology $y = n$
- A government:
  - Finances deterministic government spending $g$
  - Two tools: a distortionary labor tax $\tau$, debt $b$
  - Initial debt $b_0$

Feasibility constraint:

$$c_t + g = n_t$$
Competitive equilibrium

1. **Firm**’s static maximization: \( w = 1 \)

2. **Household**’s intertemporal maximization:

   \[ \max \sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \text{ s.t. } c_t + q_t b_{t+1} = (1 - \tau_t)n_t + b_t \forall t \]

   Two first-order conditions:

   \[ q_t = \beta U_c(t + 1)/U_c(t) \]
   \[ (1 - \tau_t) = U_c(t)/U_n(t) \]

3. **Government**’s budget constraint:

   \[ g_t + b_t = \tau_t n_t + q_t b_{t+1} \forall t \]

**Proposition (Competitive Equilibrium)**

CE Allocations are described by Implementability and Feasibility, that is:

\[ U_{c,t} c_t + U_{n,t} n_t + \beta U_{c,t+1} b_{t+1} - U_{c,t} b_t = 0, \text{ and } c_t + g_t = n_t \]
A Ramsey plan

Lucas-Stokey (primal approach, commitment)

\[
\max \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)
\]

such that

\[
U_{c,t} c_t + U_{n,t} n_t + \beta U_{c,t+1} b_{t+1} - U_{c,t} b_t = 0 \ \forall t
\]

\[
c_t + g = n_t \ \forall t
\]

Main result:

+ Taxes, multipliers, debt, are constant from period one onwards
+ If \( b_0 > 0 \), then typically \( \tau_0 < \tau_1 \).
Recursive formulation

For $t \geq 1$, two state variables (commitment):

$$ V(b, \theta) = \max_{c,n,b',\theta'} U(c, n) + \beta V(b', \theta') \quad \text{s.t.} \quad$$

$$ U_c c + U_n n + \beta \theta' b' - U_c b = 0 \quad (\Phi) $$

$$ c + g = n \quad (\lambda) $$

$$ U_c = \theta \quad (\mu) $$

In $t=0$, choice for $\theta_0 = U_{c,0}$ is free:

$$ V_0(b_0) = \max_{c_0,n_0,b_1,\theta_1} U(c_0, n_0) + \beta V(b_1, \theta_1) \quad \text{s.t.} \quad$$

$$ U_{c,0} c_0 + U_{n,0} n_0 + \beta_1 b_1 - U_{c,0} b_0 = 0 \quad (\Phi_0) $$

$$ c_0 + g = n_0 \quad (\lambda_0) $$
Some features of the solution

+ The solution is not stationary between $t = 0$ and $t = 1$:
  - Price manipulation to lower distortions: $\tau_0 < \tau_1$...
  - $q_0 > q_1$, then $q_t = q \ \forall t \geq 1$
  - Tax smoothing $\forall t \geq 1$

+ The problem is time-inconsistent:
  - Every period, incentive to $\downarrow \tau_t$... to $\uparrow q$
  - Except if...
    $$U(c, n) = c + v(n); \text{or, } b = 0; \text{or, } \Phi = 0$$

+ The only solution is stationary for $t \geq 1$:
  - $\Phi' = \Phi \ \forall b$
Markov Perfect Equilibrium

**Proposition**

A MPE is a triplet of functions \( \{B(b), N(b), V(b)\} \) such that, \( \forall b \)

\[
V(b) = \max_{c,n,b'} U(c, n) + \beta V(b') \quad \text{s.t.}
\]

\[
U_c c + U_n n + \beta U_c (N(b') - g, N(b')) b' - U_c b = 0 \quad (\Phi)
\]

\[
c + g = n \quad (\lambda)
\]

+ Guess: always an incentive to \( \downarrow \tau \) ... so \( b \) goes up to the limit?
Optimal policy? No!
What can we learn from the first-order conditions?

First-order condition, assuming differentiable $\mathcal{N}(b')$ and an interior solution for $b$:

$$\Phi' = \Phi \left( 1 + b' \left( \frac{U'_{cc} + U'_{cn}}{U'_c} \right) \frac{\partial \mathcal{N}(b')}{\partial b'} \right)$$

\[\text{(GEE)}\]

- **Stationary & interior** in two points only:
  \[\Phi = 0 \text{ (first-best) or } b = 0\]
- Can be **stationary & not interior**:
  \[b = b_{\text{max}} \text{ well-defined}\]
- Or **not stationary or not continuous**
Optimal policy: Debt

\[ B(b) \]

\[ b \]
Some features of the optimal plan

Two forces: tax smoothing, and price manipulation.

- **Upper bound** \( b = b_{\text{max}} \)
- **To the left, a flat interval**
  - **Price manipulation** motives dominate
  - GEE holds with inequality
  - Roll over as much debt as possible: \( b^\prime = b_{\text{max}} \)
- **Then, an increasing interval**
  - **Tax smoothing** motives dominate
  - GEE holds with equality
  - Higher \( b \) translates in higher \( b^\prime, \tau, \tau^\prime \)
- **And finally, a jump**
  - Because there are two peaks in the value function
  - A new local upper bound (discontinuity)
Optimal policy: Labor
Commitment vs. Non-Commitment
Conclusion

+ Time-consistency issues can be fixed using discontinuous policies
  - A "threat": the MPE is close to LS!

+ Hence time-inconsistencies related to the manipulation of interest rates may not be such a big issue...