Time-Consistent Debt

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Motivation

An optimal fiscal policy problem:

- Lucas-Stokey (1983)
- Without commitment - Markov Perfect Equilibria

Three questions:

- What is *time-inconsistent* in the Ramsey plan?
- How do policies *change*, without commitment?
- *How far* are the commitment and the non-commitment plans?

Environment

+ Time is discrete: $t = 0, 1, 2, ...$
+ A representative household:
  - Values consumption and dislikes labor $U(c, n)$
  - Savings b (complete markets)
+ A representative firm:
  - Produces output with a linear technology $y = n$
+ A government:
  - Finances **deterministic** government spending g
  - Two tools: a *distortionary labor tax* τ, *debt* b
  - Initial debt b_0

Feasibility constraint:

$$c_t + g = n_t$$

Competitive equilibrium

1. Firm's static maximization: $w = 1$
2. Household's intertemporal maximization:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, n_t) \;\; \text{s.t.} \;\; c_t + q_t b_{t+1} = (1 - \tau_t) n_t + b_t \;\forall t$$

   Two first-order conditions:

$$q_t = \beta U_c(t+1)/U_c(t)$$
$$(1 - \tau_t) = U_c(t)/U_n(t)$$

3. Government's budget constraint:

$$g_t + b_t = \tau_t n_t + q_t b_{t+1} \;\forall t$$

Proposition (Competitive Equilibrium)

CE Allocations are described by Implementability and Feasibility, that is:

$$U_{c,t} c_t + U_{n,t} n_t + \beta U_{c,t+1} b_{t+1} - U_{c,t} b_t = 0, \;\; \textit{and} \;\; c_t + g_t = n_t$$

A Ramsey plan

Lucas-Stokey (primal approach, commitment)

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$$

such that

$$U_{c,t}c_t + U_{n,t}n_t + \beta U_{c,t+1}b_{t+1} - U_{c,t}b_t = 0 \ \forall t$$
$$c_t + g = n_t \ \forall t$$

Main result:

+ Taxes, multipliers, debt, are constant *from period one onwards*
+ If $b_0 > 0$, then typically $\tau_0 < \tau_1$.

Recursive formulation

For $t \geq 1$, two state variables (commitment):

$$V(b, \theta) = \max_{c,n,b',\theta'} U(c, n) + \beta V(b', \theta') \text{ s.t.}$$

$$U_c c + U_n n + \beta\theta' b' - U_c b = 0 \qquad (\Phi)$$

$$c + g = n \qquad (\lambda)$$

$$U_c = \theta \qquad (\mu)$$

In t=0, choice for $\theta_0 = U_{c,0}$ is free:

$$V_0(b_0) = \max_{c_0,n_0,b_1,\theta_1} U(c_0, n_0) + \beta V(b_1, \theta_1) \text{ s.t.}$$

$$U_{c,0} c_0 + U_{n,0} n_0 + \beta\theta_1 b_1 - U_{c,0} b_0 = 0 \qquad (\Phi_0)$$

$$c_0 + g = n_0 \qquad (\lambda_0)$$

Some features of the solution

- The solution is not stationary between $t = 0$ and $t = 1$:
  - Price manipulation to lower distortions: $\tau_0 < \tau_1 \ldots$
  - $q_0 > q_1$, then $q_t = q \; \forall t \geq 1$
  - Tax smoothing $\forall t \geq 1$
- The problem is time-inconsistent:
  - Every period, incentive to $\downarrow \tau_t \ldots$ to $\uparrow q$
  - Except if...

$$U(c, n) = c + v(n); \text{ or, } b = 0; \text{ or, } \Phi = 0$$

- The only solution is stationary for $t \geq 1$:
  - $\Phi' = \Phi \; \forall b$

Markov Perfect Equilibrium

Proposition

A MPE is a triplet of functions $\{\mathcal{B}(b), \mathcal{N}(b), V(b)\}$ *such that,* $\forall b$

$$V(b) = \max_{c,n,b'} U(c,n) + \beta V(b') \ s.t.$$

$$U_c c + U_n n + \beta U_c(\mathcal{N}(b') - g, \mathcal{N}(b'))b' - U_c b = 0 \qquad (\Phi)$$

$$c + g = n \qquad (\lambda)$$

$+$ Guess: always an incentive to $\downarrow \tau$ ... so b goes up to the limit?

Optimal policy? No!

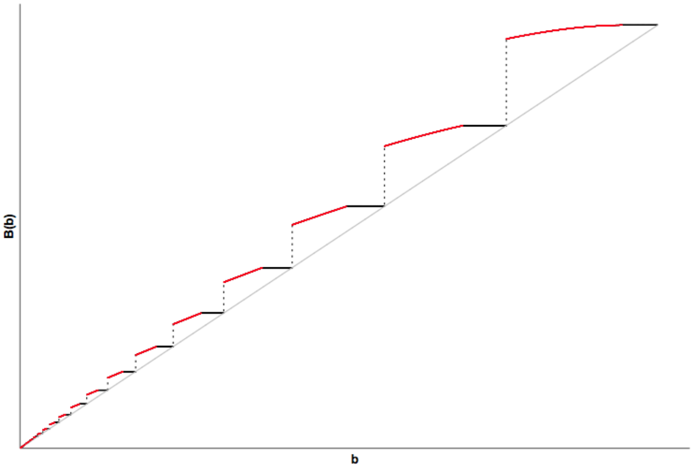
What can we learn from the first-order conditions?

First-order condition, assuming differentiable $\mathcal{N}(b')$ and an interior solution for b:

$$\Phi' = \Phi\left(1 + b'\left(\frac{U'_{cc} + U'_{cn}}{U'_c}\right)\frac{\partial\mathcal{N}(b')}{\partial b'}\right) \qquad \text{(GEE)}$$

- *Stationary & interior* in two points only:
  $\Phi = 0$ (first-best) or $b = 0$
- Can be *stationary & not interior*:
  $b = b_{max}$ well-defined
- Or *not stationary* or *not continuous*

Optimal policy: Debt

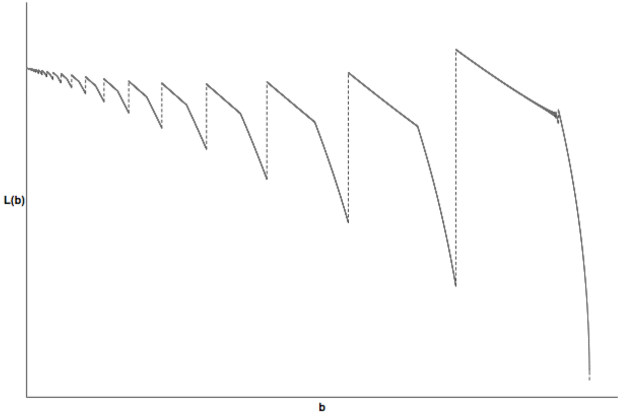


Some features of the optimal plan

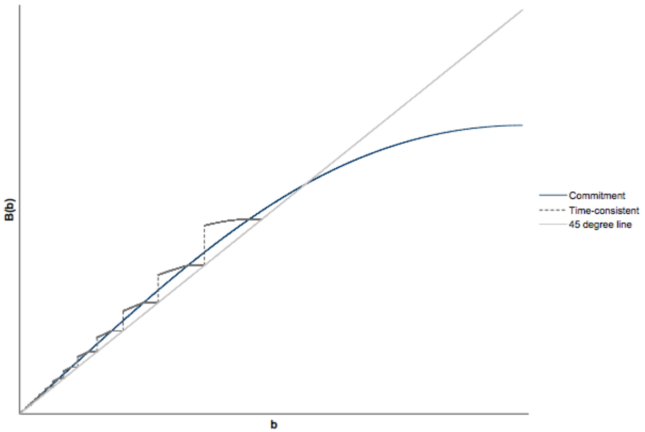
Two forces: tax smoothing, and price manipulation.

- Upper bound $b = b_{max}$
- To the left, a flat interval
  - **Price manipulation** motives dominate
  - GEE holds with inequality
  - Roll over as much debt as possible: $b' = b_{max}$
- Then, an increasing interval
  - **Tax smoothing** motives dominate
  - GEE holds with equality
  - Higher b translates in higher b', τ, τ'
- And finally, a jump
  - Because there are two peaks in the value function
  - A new local upper bound (discontinuity)

Optimal policy: Labor



Commitment vs. Non-Commitment



Conclusion

- Time-consistency issues can be fixed using discontinuous policies
  - A "threat": the MPE is close to LS!

- Hence time-inconsistencies related to the manipulation of interest rates may not be such a big issue...