Perturbation Theory in Models with Idiosyncratic Risk

David Evans

May 13, 2014

Introduction

- Want to study optimal taxation policy in an economy with incomplete markets and heterogeneous agents.
  - Specifically with uninsurable idiosyncratic Risk.
- Difficult because the distribution of wealth is now a state of the economy.
- Moreover optimal tax policy will likely depend on this distribution in complicated ways.
- Most attempts at this question resort to maximizing steady state welfare, not the same as solving a time zero optimal taxation problem.

Methodology

- We will employ a small noise expansion around a non-stochastic steady state.
- Can exploit the property of most incomplete markets economies that there are multiple non-stochastic steady states.
    - In complete markets the effective Pareto weights of individuals are constant.
    - In incomplete markets these evolve stochastically.
- Allows us to express optimal policy as Taylor expansions with respect to only the idiosyncratic shocks.
    - But allowing the coefficients to depend on both the individual state variables and the aggregate distribution.

A Simple Example

- Consider a unit mass of agents.
- A planner has planner has Pareto weights m over these agents distributed according to Γ.
- Has a single unit of the consumption good to allocate across agents, agents have log preferences over consumption
- Planner's problem is to choose a function $c(m)$ to maximize

$$\int m \log(c(m)) d\Gamma(m)$$

subject to the constraint

$$\int c(m) d\Gamma(m) = 1$$

Relabeling

- Suppose we know the solution for some $\overline{\Gamma}$, can we approximate the solution for some $\Gamma \approx \overline{\Gamma}$.
- Start by labeling each agent by $i \in (0,1)$, $\overline{m}^i$ then represents the Pareto weight of agent i under $\overline{\Gamma}$.
- The planners problem is then to choose c^i to maximize

$$\int_0^1 m^i \log c^i di$$

subject to

$$\int c^i di = 1$$

- Let λ be the multiplier on the resource constraint.

First Order Conditions

- The first order conditions associated with this problem are

$$\frac{m^i}{c^i} = \lambda$$

and

$$\int c^i di = 1$$

- We know the solution $\overline{c}^i$,$\overline{\lambda}$ for the distribution $\overline{\Gamma}$ defined by $\overline{m}^i$.
- Can we approximate the solution for a distribution Γ defined by $m^i = \overline{m}^i + \hat{m}^i$ for $\hat{m}^i$ small?

Approximating Individual Variables

- We begin by linearizing the first equation around $\overline{m}^i, \overline{c}^i, \overline{\lambda}$

$$\frac{1}{\overline{c}^i}\hat{m}^i - \frac{\overline{m}^i}{(\overline{c}^i)^2}\hat{c}^i = \hat{\lambda}$$

  where $\hat{c}^i = c^i - \overline{c}^i$ and $\hat{\lambda} = \lambda - \overline{\lambda}$
- Using $\overline{m}^i = \overline{\lambda}\overline{c}^i$ and solving for $\hat{c}^i$ we find

$$\hat{c}^i = \frac{1}{\overline{\lambda}}\hat{m}^i - \frac{\overline{c}^i}{\overline{\lambda}}\hat{\lambda}$$

Approximating Individual Variables

- We begin by linearizing the first equation around $\overline{m}^i, \overline{c}^i, \overline{\lambda}$

$$\frac{1}{\overline{c}^i}\hat{m}^i - \frac{\overline{m}^i}{(\overline{c}^i)^2}\hat{c}^i = \hat{\lambda}$$

where $\hat{c}^i = c^i - \overline{c}^i$ and $\hat{\lambda} = \lambda - \overline{\lambda}$

- Using $\overline{m}^i = \overline{\lambda}\overline{c}^i$ and solving for $\hat{c}^i$ we find

$$\hat{c}^i = \frac{1}{\overline{\lambda}}\hat{m}^i - \frac{\overline{c}^i}{\overline{\lambda}}\hat{\lambda}$$

- Note that $\hat{c}^i$ is directly affected by $\hat{m}^i$ and affected by $\hat{m}^j$, for $j \neq i$, through changes in the aggregate variable $\hat{\lambda}$

Approximating Aggregate Variables

- If we linearize the resource constraint we obtain

$$\int_0^1 \hat{c}^i di = 0$$

- This is going to pin down $\hat{\lambda}$ when substitute in our expression for $\hat{c}^i$

$$\frac{1}{\bar{\lambda}} \int_0^1 \hat{m}^i di - \frac{\hat{\lambda}}{\bar{\lambda}} \int_0^1 c^i di = 0$$

- This simplifies to

$$\hat{\lambda} = \int_0^1 \hat{m}^j dj$$

- The combination of all the deviations $\hat{m}^j$ determines $\hat{\lambda}$

Features

- Individual variables change in response to changes in their own individual state, m^i and with respect to the aggregate state through aggregate variables.
- The slopes may depend on the individual state:
  - Suppose $\hat{m}^i = 0.1$ for all i and $\overline{\lambda} = 1$
  - Then $\hat{\lambda} = 0.1$ and

$$\hat{c}^i = 0.1\left(1 - \overline{c}^i\right)$$

  - Consumption for individuals with low Pareto weight responds more than consumption of individuals with high Pareto weight.
- Matches response of global solution to increasing each Pareto weight by small amount.

DSGE

- Let's start with a procedure we are all familiar with.
- The first order conditions for a DSGE model can be put in the form
$$\mathbb{E}_t F(y_t, y_{t+1}, \epsilon_t, \xi_{t-1}|q) = 0 \tag{1}$$
- Here y_t are controls, ξ_{t-1} are the state variables, ϵ_t are the shocks and q is a factor which scales the variance of the shock
- For example, the log productivity process could follow
$$a_t = \rho a_{t-1} + q\epsilon_t$$
- We want to find/approximate policy rules $y_t = y(\epsilon_t, \xi_{t-1}|q)$ and $\xi_t = \xi(\epsilon_t, \xi_{t-1}|q)$ that solve this system of equations

Perturbation Theory and DSGE

- Begin by finding the non-stochastic steady state $\overline{\xi}, \overline{y}$ such that $\overline{\xi} = \xi(\epsilon, \overline{\xi}|0)$ and
$$F(\overline{y}, \overline{y}, \epsilon, \overline{\xi}|0) = 0$$
- The idea then is to recover the optimal policy functions using a truncated taylor expansion around the non-stochastic steady state
    - Need derivatives with respect to states, shocks and the volatility parameter q.
- Compute the derivatives by applying the implicit function theorem around the non-stochastic steady state.

Idiosyncratic Risk

- With idiosyncratic risk we divide the equations governing the economy into individual constraints

$$\mathbb{E}_t F(y_t, \mathbb{E}_{t-1} y_t, Y_t, y_{t+1}, \epsilon_t | \xi_{t-1}, q, \Gamma_t) = 0 \text{ for all } \xi_{t-1}$$

and aggregate constraints

$$\int G(y_t, , Y_t, \epsilon_t | \xi_{t-1}, q, \Gamma_t) d\Gamma_t(\xi_{t-1}) dF_\epsilon(\epsilon_t) = 0$$

- Here Γ_t is the joint distribution over the idiosyncratic state variables ξ_{t-1} and Y_t are the aggregate control variables.
- In addition, we've included $\mathbb{E}_{t-1} y_t$ terms to capture measurability constraints in this economy.

Non-Stochastic Steady States

- As with the DSGE model we want to approximate the policy rules $y_t = y(\epsilon_t|\xi_{t-1}, q, \Gamma_t)$ and $Y_t = Y(\Gamma_t)$ that solve that system of equations.
    - As ξ_t can be chosen to be a subset of y_t; the policy rules for y_t induces a law of motion $\xi_t = \xi(\epsilon_t|\xi_{t-1}, q, \Gamma_t)$
    - Which allows us to construct a law of motion $\Gamma_{t+1} = \Gamma(\Gamma_t|q)$
- The first step is to choose state variables such that the law of motion for ξ_t satisfies

$$\xi_{t-1} = \xi(\epsilon_t|\xi_{t-1}, 0, \Gamma_t)$$

for all Γ_t,ξ_{t-1}.

- This implies that $\Gamma(\Gamma_t|0) = \Gamma_t$ for all Γ_t

Non-Stochastic Steady States

- As with the DSGE model we want to approximate the policy rules $y_t = y(\epsilon_t|\xi_{t-1}, q, \Gamma_t)$ and $Y_t = Y(\Gamma_t)$ that solve that system of equations.
  - As ξ_t can be chosen to be a subset of y_t; the policy rules for y_t induces a law of motion $\xi_t = \xi(\epsilon_t|\xi_{t-1}, q, \Gamma_t)$
  - Which allows us to construct a law of motion $\Gamma_{t+1} = \Gamma(\Gamma_t|q)$
- The first step is to choose state variables such that the law of motion for ξ_t satisfies

$$\xi_{t-1} = \xi(\epsilon_t|\xi_{t-1}, 0, \Gamma_t)$$

  for all Γ_t,ξ_{t-1}.
- This implies that $\Gamma(\Gamma_t|0) = \Gamma_t$ for all Γ_t
- Any distribution $\overline{\Gamma}$ is a viable non-stochastic steady state.

Linear Approximation

- As before choose any $\overline{\Gamma}$ as a candidate steady state around which we will approximate the policy rules.
- For ease of notation, label each agent by $i \in (0, 1)$
- Let $\overline{\xi}^i$ be the steady state value of ξ for agent i.
- The deviation for agent i from the steady state is $\hat{\xi}^i_{t-1} = \xi^i_{t-1} - \overline{\xi}^i$, which describes the current aggregate state Γ_t.
- We begin with analyzing the linear approximation to $\hat{Y}_t = Y_t - \overline{Y}$, this will be the sum of the linear contributions of $\hat{\xi}^i_{t-1}$

$$\hat{Y}_t = \int dY^i_\xi \hat{\xi}^i_{t-1} di$$

Linear Approximation

- For

$$\hat{Y}_t = \int dY_\xi^i \hat{\xi}_{t-1}^i di$$

  dY_ξ^i represents the contribution of $\hat{\xi}_{t-1}^i$ to get aggregate controls.

- Note that it is implicitly a function of both the aggregate and individual steady state variables: $dY_\xi^i(\overline{\xi}^i, \overline{\gamma})$.
- The linear approximation to $\hat{y}_t^i = y_t^i - \overline{y}^i$ will be a combination of the linear contributions of $\hat{\xi}_{t-1}^i$,$q\epsilon_t^i$ and $\hat{\xi}_{t-1}^j$ for $j \neq i$. Thus we can write the linear approximation to $\hat{y}_t^i$ as

$$\hat{y}_t^i = dy_\xi^i \hat{\xi}_{t-1}^i + dy_\epsilon^i q\epsilon_t^i + \int dy_\Gamma^{i,j} \hat{\xi}_{t-1}^j dj$$

Linear Approximation

- Currently our expression for $\hat{y}^i_t$ is unwieldy, $dy^{i,j}_\Gamma$ depends on steady state values for both i and j.
- This would be lots of terms to compute.
- We can simplify this further by noting that agent j's state will only effect i though changes in aggregate variables. Hence,

$$\hat{y}^i_t = dy^i_\xi \hat{\xi}^i_{t-1} + dy^i_\epsilon q \epsilon^i_t + dy^i_Y \hat{Y}_t$$

- Note that we can then write $dy^{i,j}_\Gamma = dy^i_Y dY^j_\xi$.

Future Variables

- Our individual equations depend on future realizations of $\hat{y}^i_{t+1}$ which in turn depend on $\hat{\xi}^i_t$ and $\hat{Y}_{t+1}$
- For the law of motion of the state variables we can show that

$$\hat{\xi}^i_t = \hat{\xi}^i_{t-1} + qd\xi^i_\epsilon \epsilon^i_t$$

- We can use this law of motion to forecast the aggregate variables next period

$$\hat{Y}_{t+1} = \int dY^i_\xi \xi^i_t di = \int dY^i_\xi (\hat{\xi}^i_{t-1} + d\xi^i_\epsilon q\epsilon^i_t) di = \hat{Y}_t$$

- As well as the forecast of future controls

$$\mathbb{E}_t \hat{y}^i_{t+1} = dy^i_\xi \hat{\xi}^i_{t-1} + dy^i_Y \hat{Y}_t + dy^i_\xi d\xi^i_\epsilon q\epsilon^i_t$$

- Use these formuals and the differentiation of the constraints to compute linear approximation terms.

Taking Stock

- A reasonable question at this point is why are we going through the trouble of computing dY^i_ξ and dy^i_ξ?
- The choice of $\overline{\Gamma}$ is arbitrary. If we choose $\overline{\Gamma}$ to be equal to Γ_t then $\hat{\xi}^i_{t-1} = 0$ for all i.
- Under a linear approximation certainty equivalence applies, the variance of the stochastic shock does not enter the policy rules.
  - We saw that the linear forecast of $\hat{Y}_{t+1}$ which was $\hat{Y}_t$
- With a quadratic approximation we introduce uncertainty but, because our agents our forward looking, we need the dY^i_ξ terms to compute the effects of the uncertainty tomorrow on the allocations today.

Quadratic approximation: Aggregate Terms

- For the most part computing the second order terms is a just a more complicated version of the linearization computations. There are a few differences
- We need to keep track of the linear contributions to the aggregate state, thus it is useful to create the stacked variable

$$\hat{S}_t^i = \begin{pmatrix} \hat{\xi}_{t-1}^i \\ \int dY_\xi^i \hat{\xi}_{t-1}^i di \end{pmatrix}$$

- With that definition we can express $\hat{Y}_t$ as

$$\hat{Y}_t = \int \left[dY_\xi^i \hat{\xi}_{t-1}^i + \frac{1}{2} (\hat{S}_t^i)' dY_{SS}^i \hat{S}_t^i \right] di + \frac{1}{2} dY_{q^2} q^2$$

Quadratic Approximation: Individual Terms

- Similarly we can expand the our approximation of the individual controls as follows

$$\hat{y}^i_t = dy^i_\xi \hat{\xi}^i_{t-1} + dy^i_\epsilon q\epsilon^i_t + dy^i_Y \hat{Y}_t + \frac{1}{2}\Big[(\hat{S}^i_t)' dy^i_{SS}\hat{S}^i_t + (\hat{S}^i_t)' dy^i_{S\epsilon} q\epsilon^i_t + (q\epsilon^i_t)' dy^i_{\epsilon S}\hat{S}^i_t + dy^i_{\epsilon\epsilon} q^2 (\epsilon^i_t)^2 + dy^i_{q^2} q^2\Big]$$

- The key term here is the $dy^i_{q^2} q^2$ which captures how the change in the distribution of individual states tomorrow will affect aggregate variables

$$\hat{Y}_{t+1} = \hat{Y}_t + \frac{1}{2}\left(\int \left[dY^i_\xi I^\xi_y (dy^i_{\epsilon\epsilon} + dy^i_{q^2}) + (I^\xi_y dy^i_\epsilon)' dY^i_{\xi\xi} I^\xi_y dy^i_\epsilon\right] di\right) q^2$$

Implentation

Once we have this formalism simulating with these approximations is straightforward:

1. Begin with a distribution Γ_t. Use this to solve for the non-stochastic steady state. Thus $\hat{\xi}^i_t = 0$ for all i.
2. The policy rules can be approximated as

$$y^i_t = \overline{y}^i + dy^i_\epsilon q \epsilon^i_t + \frac{1}{2}\left[dy^i_{\epsilon\epsilon} q^2 (\epsilon^i_t)^2 + dy^i_{q^2} q^2 + dy^i_Y dY_{q^2} q^2 \right]$$

and

$$Y_t = \overline{Y} + \frac{1}{2} dY_{q^2} q^2$$

3. Use the policy rules to construct a new distribution Γ_{t+1} (For example: approximate Γ_t using N agents and draw Γ_{t+1} via policy rules)
4. Go back to step 1.

Features

- What may be hidden in the subscript i, is that the coefficients in these approximations depend on both the current aggregate distribution Γ_t and the individual state ξ^i_{t-1}:

$$y(\epsilon^i_t|\xi^i_{t-1},\Gamma_{t-1}) \approx \overline{y}(\xi^i_{t-1},\Gamma_{t-1}) + dy_\epsilon(\xi^i_{t-1},\Gamma_{t-1})q\epsilon^i_t + \frac{1}{2}\left[dy_{\epsilon\epsilon}(\xi^i_{t-1},\Gamma_{t-1})(\epsilon^i_t)^2 + dy_{q^2}(\xi^i_{t-1},\Gamma_{t-1})\right]q^2$$

and

$$Y(\Gamma_t) \approx \overline{Y}(\Gamma_t) + \frac{1}{2}dY_{q^2}(\Gamma_t)q^2$$

- Introduces a lot of non-linearity without needing interpolation, computational costs increase linearly with each additional individual state.
- No need to find an ergodic steady state!

Application

- As an example, we'll use this method to solve Ramsey taxation problem with uninsurable idiosyncratic labor risk.
- This approximation allows us to approximate the time-0 Ramsey plan.
- As opposed to finding the level of labor taxation that maximizes steady state welfare.
- We'll calibrate the model in two ways: iid productivity shocks and productivity shocks that follow a random walk.
- Long run taxation implications will be drastically different.

Environment

- There are a unit mass of ex-ante identical agents who have preferences over consumption and labor

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i)$$

- Output is linear in labor. Aggregate productivity is constant and normalized to 1. Labor markets are perfectly competitive, so an agent's pre-tax wage is her productivity: $\exp(e_t^i)$.
- Productivity is AR(1) in logs

$$e_t^i = (1-\nu)\bar{e} + \nu e_{t-1}^i + q\epsilon_t^i$$

where ϵ_t^i is iid $\mathcal{N}(0,1)$.
- Agents cannot perfectly insure against labor income shocks but can borrow and lend using a one-period risk free bond and natural borrowing limits.

Environment: Government

- Government is modeled by a benevolent planner with a utilitarian objective function.
- Has access to a proportional labor tax τ_t and lump sum transfers T_t to redistribute resources across agents.
- Can issue government debt B_t at the risk-free rate R_t
- The governments budget constraint is then

$$R_{t-1}B_{t-1} + T_t = \tau_t \int_i e_t^i l_t^i di + B_t$$

- While the agent's budget constraint is

$$R_{t-1}b_{t-1}^i + (1-\tau_t)e_t^i l_t^i + T_t = c_t^i + b_t^i$$

Equilibrium

- Given a sequence of government policies $\{\tau_t, T_t\}$ and initial asset and productivity distribution $\{b_0^i, e_0^i\}$, a competitive equilibrium is a sequence of price $\{R_t\}$ and policy functions $c_t(e^{i,t})$, $l_t(e^{i,t})$ and $b_{t+1}(e^{i,t})$ such that
  1. Conditional on prices and government policy the policies solve the households maximization problem.
  2. The aggregate resource constraint is satisfied

$$\int c_t^i di = \int l_t^i \exp(e_t^i) di$$

- The Ramsey taxation problem is to find the competitive equilibrium that maximizes the utilitarian objective function of the planner.

Equilibrium Conditions

Optimality for the household can be summarized by the FOC governing labor-leisure choice

$$-u^i_{l,t} = (1-\tau_t)u^i_{c,t}\exp(e^i_t)$$

the inter-temporal Euler equation

$$u^i_{c,t} = \beta R_t \mathbb{E}_t u^i_{c,t+1}$$

and the budget constraint

$$R_{t-1}b^i_{t-1} + (1-\tau_t)\exp(e^i_t)l^i_t + T_t = c^i_t + b^i_t$$

Equilibrium Conditions

- We can substitute the two FOC's into the budget constraint and multiply by $u^i_{c,t}$ to obtain

$$\frac{x^i_{t-1}u^i_{c,t}}{\beta\mathbb{E}_{t-1}u^i_{c,t}} = u^i_{c,t}(c^i_t - T_t) + u^i_{l,t}l^i_t + x^i_t$$

  where $x^i_{t-1} = u^i_{c,t-1}b^i_{t-1}$.

- Let m^i_t be the effective Pareto weight at time t then

$$\alpha^2_t = m^i_t u^i_{c,t}$$

  for all i for some α^2_t.

- The inter-temporal Euler equation is then satisfied if the following constraint holds

$$\alpha^1_t = m^i_t \mathbb{E}_t u^i_{c,t+1}$$

  for $\alpha^1_t = \frac{\alpha^2_t}{\beta R_t}$.

Ramsey Problem

$$\max_{c_t^i, l_t^i, x_t^i, m_t^i, \tau_t, T_t, \alpha_t^1, \alpha_t^2} \sum_t \beta^t \int u(c_t^i, l_t^i) di$$

subject to the individual constraints for $t \geq 1$

$$\begin{aligned}
\mu_t^i : \quad & \frac{x_{t-1}^i u_{c,t}^i}{\beta \mathbb{E}_{t-1} u_{c,t}^i} = u_{c,t}^i (c_t^i - T_t) + u_{l,t}^i l_t^i + x_t^i \\
\rho_t^{1,i} : \quad & \alpha_t^1 = m_t^i \mathbb{E}_t u_{c,t+1}^i \\
\rho_t^{2,i} : \quad & \alpha_t^2 = m_t^i u_{c,t}^i \\
\phi_t^i : \quad & -u_{l,t}^i = (1 - \tau_t) u_{c,t}^i e_t^i
\end{aligned}$$

Ramsey Problem

and the aggregate constraints

$$\eta_t : \qquad \int m_t^i di = 1$$

$$\lambda_t : \qquad \int l_t^i e_t^i di = \int c_t^i di$$

and the law of motion for the idiosyncratic productivity process

$$e_t^i = (1-\nu)\bar{e} + \nu e_{t-1}^i + q\epsilon_t^i$$

- Three individual state variables $(m_{t-1}^i, \mu_{t-1}^i, e_{t-1}^i)$

FOC

- Their are multiple first order conditions for this problem, I will highlight a few of them which are of particular interest.
- The FOC with respect to x^i_{t-1} gives

$$\mu^i_{t-1} = \frac{\mathbb{E}_{t-1} u^i_{c,t} \mu^i_t}{\mathbb{E}_{t-1} u^i_{c,t}}$$

  the marginal value of assets to agent i follows a twisted martingale.
- The FOC with respect to τ_t gives

$$\int \phi^i_t u^i_{c,t} e^i_t di = 0$$

- The FOC with respect to T_t gives

$$\int \mu^i_t u^i_{c,t} di = 0$$

Ricardian Equivalence

The first order condition with respect to T_t can be rewritten as

$$\int \frac{\mu_t^i}{m_t^i} = 0$$

Our equation governing asset pricing can also be rewritten as

$$\alpha_{t-1}^1 \frac{\mu_{t-1}^i}{m_{t-1}^i} = \alpha_t^2 \mathbb{E}_{t-1} \frac{\mu_t^i}{m_t^i}$$

We can integrate both sides and exploit the fact that we have a continuum of agents to get

$$\alpha_{t-1}^1 \int \frac{\mu_{t-1}^i}{m_{t-1}^i} di = \alpha_t^2 \int \frac{\mu_t^i}{m_t^i} di$$

Thus if $\int \frac{\mu_0^i}{m_0^i} di = 0$ holds at time zero the FOC w.r.t T_t will be satisfied for all T.

Rough Calibration

- Aggregate productivity was normalized to 1.
- The discount rate β was set to 0.95
- The period utility function was taken to be the form

$$U(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$$

  with $\sigma = 2.$ and $\gamma = 2$
- We solved the model under two different regimes:

$$e_t^i = q\epsilon_t^i$$

  and

$$e_t^i = e_{t-1}^i + q\epsilon_t^i$$

- In both cases q was chosen such that the innovations $\sum_{t=0}^{\infty} \beta^t e_t^i$ were the same.

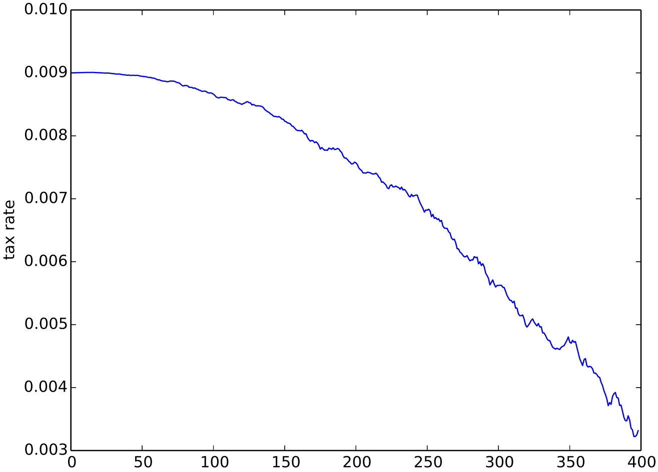

Figure : Taxes in the iid economy.

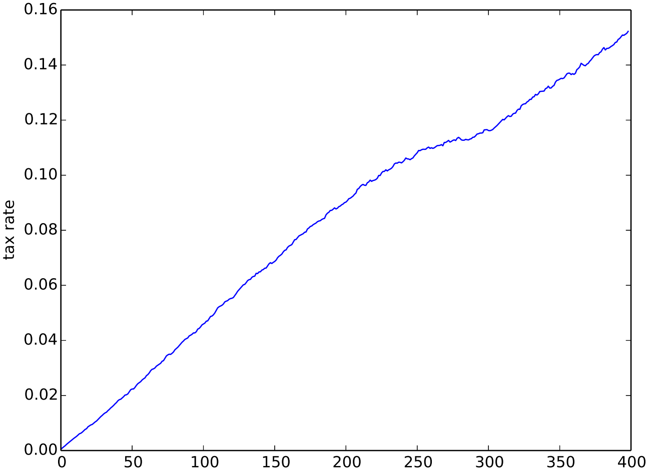

Figure : Taxes in the unit root economy.

Why the different policies?

- Both economies feature the same size shocks to the present value of of lifetime productivity.
- Why do they feature such different time paths to the tax rate τ_t?

Why the different policies?

- Both economies feature the same size shocks to the present value of of lifetime productivity.
- Why do they feature such different time paths to the tax rate τ_t?
- The planners only ability to redistribute resources is through labor taxation.
- In the iid case agents receive a large income shock in 1 period, they smooth their consumption through time via savings
  - The government is only able to directly redistribute that income shock in the period it arrives, not after.
- In the unit root case agents don't need to save as much as they've received a permanent income shock.
  - The government can raise taxes a small amount forever to redistribute that additional income.
  - As more of these shocks accrue overtime tax rates will continue to rise

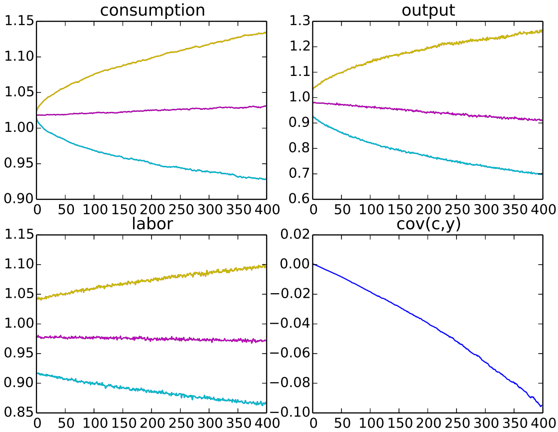

Figure : Inequality in the iid economy. The figure plots the quantiles for consumption, pre-tax labor earnings, labor and the covariance between consumption and pre-tax labor earnings

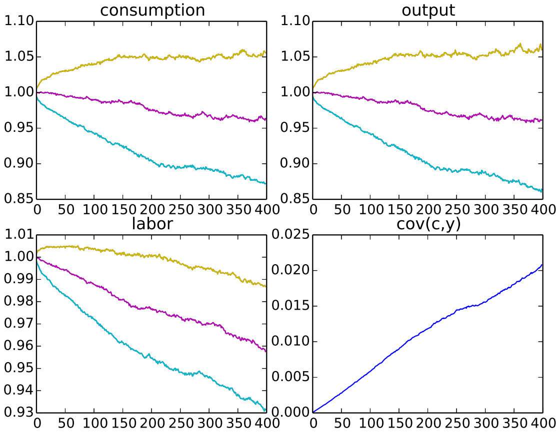

Figure : Inequality in the unit root economy. The figure plots the quantiles for consumption, pre-tax labor earnings, labor and the covariance between consumption and pre-tax labor earnings

Cov(c,y)

- The iid case featured a covariance between pre-tax income and consumption that was decreasing in time
  - Inequality was entirely driven by previous income shocks which have no relevance to current productivity.
  - The wealth effect then implies that agents with high consumption (lot's of assets) will work less.
- The best way to redistribute resources is to start subsidizing labor income via lump sum taxation.
- In the unit root case features agents with high consumption will also have high productivity, and hence, high income.
- In this case the best way to redistribute resources is via proportional labor taxation.

Computing The Linearization

- We compute the linearization using the implicit function theorem. Differentiation F around the steady state values we obtain

$$F^i_y \hat{y}^i_t + F^i_{Ey}\mathbb{E}_{t-1}\hat{y}^i_t + F^i_Y \hat{Y}_t + F^i_{y'}\mathbb{E}_t \hat{y}^i_{t+1} + F^i_\epsilon q\epsilon^i_t + F^i_\xi \hat{\xi}^i_{t-1} = 0$$

- The superscript i denotes the evaluation of the derivatives at $\overline{\xi}^i$ and $\overline{y}^i$.
- We can then plug in our formulas for $\hat{y}^i_t$,$\hat{y}^i_{t+1}$ and solve for like terms

$$dy^i_\xi = -(F^i_y + F^i_{Ey} + F^i_{y'})^{-1} F^i_\xi \hat{\xi}^i_{t-1} \tag{2a}$$

$$dy^i_\epsilon = -(F^i_y + F^i_{y'} dy^i_\xi I^\xi_y)^{-1} F^i_\epsilon \tag{2b}$$

$$dy^i_Y = -(F^i_y + F^i_{Ey} + F^i_{y'})^{-1} F^i_Y \tag{2c}$$

Aggregate Terms

- As with the simple example we use the aggregate constraints determine the linear coefficients for the linearization of the aggregate variable

$$\int \left[G_y^i \hat{y}_t^i + G_Y^i \hat{Y}_t + G_\xi^i \hat{\xi}_{t-1}^i \right] di = 0$$

- We can then substitute in our expression for $\hat{y}_t^i$ to get

$$dY_\xi^i = -\left(\int G_y^i dy_Y^i + G_Y^i di \right)^{-1} \left(G_y^i dy_\xi^i + G_\xi^i \right)$$

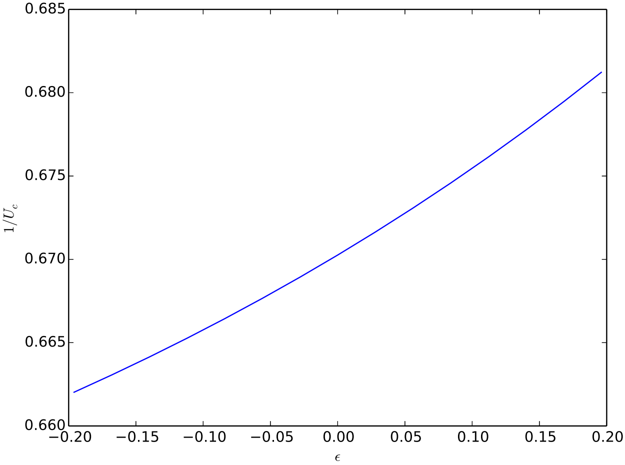

Figure : An example policy function for m_t using global solution methods for a fixed R_t