Self-Fulfilling Debt Crises

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Mexican Crisis: 1994-1995

- Fear of government default led to the government being unable to issue new debt...
- ...in turn confirming the fear of default! (until the US intervened)

Mexican *fiscal fundamentals* were stable and comparable to those of healthy governments.

**Key Feature**: average maturity of Mexican debt had become very short.
Self-Fulfilling Debt Crises

A Simple Example

Risk-neutral investors, deep pockets, no constraints

\[ q = \beta \mathbb{E}[H] \]

price of bond \hspace{1cm} expected haircut

Government budget constraints:

\[ t = 0 : \; qB = D_0 \]
\[ t = 1 : \; 0 = D_1 + HB \]

Then

\[ H = -q \frac{D_1}{D_0} \]

So \( H \) depends on its own expected value!
This Paper

Main questions:

1. How can debt crises such as the Mexican one arise in the context of a rational expectations DSGE?
2. What should optimal policy responses be, from a benevolent government under the threat of such crises?

Preview of the results:

1. **Sunspot equilibria**: aggregate outcomes determined by the beliefs of market participants for certain regions of the fundamentals;
2. Governments have incentives to reduce debt and abandon the “crisis zone” since that generates booms and reductions in yields;
3. Increasing debt maturity shrinks the crisis zone (Mexican problem);
4. Usual credibility enhancement policy remedies may be counterproductive.
Environment

- Time is discrete, infinite horizon;
- Single good, consumed or saved as capital;
- Agents:
  1. Consumers (eat, save, produce);
  2. International bankers (eat, buy government debt);
  3. Government (taxes, spends, issues debt, may default).

Timing in each period:

1. Sunspot variable $\zeta_t$ is realized; the aggregate state of the economy is
   \[ s_t = (B_t, K_t, a_{t-1}, \zeta_t) \]
2. Government chooses $B_{t+1}$, taking $q(s_t, B_{t+1})$ as given
3. Bankers choose $b_{t+1}$, taking $q_t$ as given
4. Government chooses how much to consume $g_t$ and whether to default or not $z_t \in \{0, 1\}$
5. Households choose $c_t$, $k_{t+1}$, taking $a_t$ as given
Households and Bankers

Households solve:

\[
V_c(k, s, B', g, z) = \max_{c, k'} \{ c + v(g) + \beta \mathbb{E} V_c(k', s', B'(s'), g', z') \}
\]

s.t.

\[
c + k' \leq (1 - \theta) a(s, z) f(k)
\]
\[
c, k' \geq 0
\]
\[
s' = [B', K'(s, B', g, z), a(s, z), \zeta']
\]
\[
g' = g(s', B(s'), q(s', B'(s'))]
\]
\[
z' = z(s', B'(s'), q(s', B'(s'))]
\]

Bankers solve:

\[
V_b(b, s, B') = \max_{b'} \{ \bar{x} + z[s, B', q(s, B')] b - q(s, B') b' + \beta \mathbb{E} V_b(b', s', B'(s')) \}
\]

s.t.

\[
q(s, B') b' \leq \bar{x}
\]
\[
b' \geq -A
\]
\[
s' = [B', K'(s, B', g, z), a(s, z), \zeta']
\]
Government

First, chooses how much debt to issue

\[ V_g(s) = \max_{B'} \left\{ c(K, s, B', g, z) + v(g) + \beta \mathbb{E} V_g(s') \right\} \]

s.t.

\[ g = g[s, B', q(s, B')] \]
\[ z = z[s, B', q(s, B')] \]
\[ s' = [B', K'(s, B', g, z), a(s, z), \zeta'] \]

Then, how much to consume and whether to default or not

\[ \max_{g \geq 0, z \in \{0,1\}} \left\{ c(K, s, B', g, z) + v(g) + \beta \mathbb{E} V_g(s') \right\} \]

s.t.

\[ g + zB \leq \theta a(s, z)f(K) + qB' \]
\[ s' = [B', K'(s, B', g, z), a(s, z), \zeta'] \]
Equilibrium

A recursive equilibrium is:

- Value functions: \( V_c, V_b, V_g \)
- Policy functions: \( c, k', b', \) and \( B', g, z \)
- Price function \( q \)
- Law of motion for capital \( K' \)

such that:

- Given \( B', g, z, V_c \) solves the household problem and \( (c, k') \) are the optimizing policies.
- Given \( B', q, z, V_b \) solves the banker’s problem and \( B' \) chosen by the government solves the problem when \( b = B \).
- Given \( q, c, K', g, z, V_g \) solves the government’s (first) problem, and \( B' \) is the optimizing policy. Also, given \( c, K', V_g, B', (g, z) \) solve the government’s second problem.
- \( B'(s) \in b'(B, s, B') \)
- \( K'(s, B', g, z) = k'(K, s, B', g, z) \)
The Crisis Zone

Self-fulfilling crises arise when there are two possible equilibrium outcomes.

- Possible for certain values of the fundamentals \((B, K)\);
- Sunspot variable determines which outcome ensues.

Let \(\pi \in [0, 1]\) parametrize the probability of a crisis.

Define the **crisis zone** as

\[
\bar{b}(K) \leq B \leq \bar{B}(K, \pi)
\]

- If \(B \leq \bar{b}(K)\), no crisis occurs \(\forall \zeta\);
- If \(B \geq \bar{B}(K, \pi)\), a crisis always occurs \(\forall \zeta\).
- If \(B \in CZ\), bankers predict default if \(\zeta \leq \pi\) and repayment if \(\zeta > \pi\).
Private Agents

- **Bankers** - Depending on their beliefs, can offer to buy up to $\bar{x}$ in value of govt debt at prices

\[
q(B') = \begin{cases} 
\beta & \text{if } B' \leq \bar{b}(k^n) \text{ and } z = 1 \\
\beta(1 - \pi) & \text{if } \bar{b}(k^n) \leq B' \leq \bar{B}(k^\pi, \pi) \text{ and } z = 1 \\
0 & \text{otherwise}
\end{cases}
\]

- **Households** - Due to risk-neutrality, problem is easy to solve

\[
K'(B') = \begin{cases} 
kn & \text{if } B' \leq \bar{b}(k^n) \text{ and } z = 1 \\
k^\pi & \text{if } \bar{b}(k^n) \leq B' \leq \bar{B}(k^\pi, \pi) \text{ and } z = 1 \\
k^d & \text{otherwise}
\end{cases}
\]

where $k^n > k^\pi > k^d$. 
Government Incentives

After $B'$ is sold at a positive price, there is no default iff

**Participation Constraint**: $V^g_n(s, B', q) \geq V^d_g(s, B', q)$

For $q = \beta(1 - \pi)$, this constraint determines $\bar{B}(K, \pi)$.

For a crisis to be possible (and *credible*), the government must default whenever it is unable to sell debt at a positive price

**No-Lending Condition**: $V^d_g(s, 0, 0) > V^n_g(s, 0, 0)$

This determines $\bar{b}(K)$.

Whenever the above inequalities are satisfied, a crisis zone exists.
Policy outside the Crisis Zone

If $B_t \leq \bar{b}(k^n)$, can show that

$$g_t = g_{t+1}$$
$$B_t = B_{t+1}$$

as well as $K' = k^n$ and $q = \beta$ for all $t$.

No Crisis Zone is an Absorbing State

If $B_t > \Bar{B}(K, \pi)$, the PC is violated and the government defaults.
Policy in the Crisis Zone

What if $B_0 \in [\bar{b}(K_0), \bar{B}(K_0, \pi)]$?

1. Default immediately;
2. Never run the debt down (stationary policy);
3. Run the debt down in $T < \infty$ periods;

How does this option work? The government will never run the debt below $\bar{b}(k^n) > \bar{b}(k^{\pi})$, so:

1. For $t \in \{0, \ldots, T - 2\}$, spending constant at $g^T(B_0)$ and consumption at $c^{\pi}$; probability of a crisis equal to $\pi$
2. At $T - 1$, debt set to $B_T = \bar{b}(k^n)$, consumption equal to $c^n$ if no crisis occurs;
3. From $T$ onwards, both government spending and investment increase and the economy leaves the crisis zone permanently.
Characterizing the Zones

General characterization is difficult since one must consider all possible \((K_0, B_0)\) combinations and government policies will generically not be stationary in the crisis zone.

Can show that \(\exists B^s(\pi)\) for which debt is stationary in the crisis zone (PC in equality):

1. If \(B_0 > B^s(\pi)\), PC binds and debt is immediately reduced;
2. If \(B_0 < B^s(\pi)\), debt runs down in \(T(B_0) < \infty\).
Sunspots and Self-Fulfilling Crises

Proposition

For any $K_0$ and $B_0 + \theta f(K_0) \leq B^s(\pi) + \theta f(k^\pi)$, let $V_g^T$ denote the value of reducing the debt to $\bar{b}(k^n)$ in $T$ periods. Then, a $T \in \{1, 2, \ldots, \infty\}$ that maximizes $V_g^T$ exists and

1. If $K_0 \geq k^\pi$, as $B_0$ increases, $T(B_0)$ passes critical points where it increases by one period.

2. If $K_0 < k^\pi$, the debt may increase in the first period but afterwards follows the characterization in 1.

Intuition for 2: If $B_0$ is very high, the PC binds in period 0. The government either defaults or smooths $g_t$ and sets $B_1 < \bar{B}$. 
Main Result

For any probability $\pi > 0$ for which there exists a nonempty crisis zone $\bar{b}(k^n) < B \leq \bar{B}(k^\pi, \pi)$, there exists an equilibrium in which:

1. If $\bar{b}(K_0) \leq B_0 \leq \bar{B}(K_0, \pi)$, then a crisis occurs with probability $\pi$ in the first period and every subsequent period in which $B > \bar{b}(k^n)$.
   - If $B_0 + \theta f(K_0) \leq B^s(\pi) + \theta f(k^\pi)$, optimal government policy involves running down the debt to $\bar{b}(k^n)$ in $T(B_0)$ periods.
   - If $B_0 + \theta f(K_0) > B^s(\pi) + \theta f(k^\pi)$, the government starts running down the debt in at most two periods.

2. If $B_0 \leq \bar{b}(k^n)$,
   - If $K_0 \geq k^n$, the economy is stationary in the no-crisis zone.
   - If $K_0 < k^n$ and $B_1 \leq \bar{b}(k^n)$, same as above.
   - If $K_0 < k^n$ and $B_1 > \bar{b}(k^n)$, everything proceeds as in 1.

3. If $B_0 > \bar{B}(K_0, \pi)$, the only outcome is default.
Debt Trajectories

- **DEFAULT ONLY ZONE**
  - $q = 0$, $k = k^d$

- **CRISIS ZONE**
  - $q = (1-\pi)\beta$
  - $k = k^\pi$

- **NO CRISIS ZONE**
  - $q = \beta$, $k = k^n$

Binding Participation Constraint

- Points: $b_1, b_2, b_3, b_4, b_5$
Some Results

- General results depend on the initial level of capital.
- For $B$ close to $\bar{b}(k^n)$, the government may be able to raise more revenue by selling more debt as

$$\beta(1 - \pi)B < \beta \bar{b}(k^n)$$

for $\pi$ large enough.

- As $\pi \to 1$, this region increases and eventually encompasses the entire crisis zone: transition out of the crisis zone occurs within one period for any $(B_0, K_0)$.
Extensions

- **Domestically Initiated Crises** - Crises may also be triggered by self-fulfilling fears of domestic investors: by setting $k^d < k^n$, they reduce tax revenues and may trigger default.

- **Temporary Cost of Default** - If $\alpha < 1$ for a finite horizon and the government may regain access to international markets, the set of equilibria expands. Sunspot equilibria in which the probability of a crisis depends on the history of past defaults become possible.
Policy Implications

- **Credibility** - Suppose the government sets $\downarrow \alpha$ as a (limited) commitment device. This “relaxes” the crisis region $\uparrow \bar{b}(K), \uparrow \bar{B}(K, \pi)$, but does not eliminate it, potentially worsening the consequences of a crisis.

- **Maturity** - By lengthening the maturity structure, the government can increase $\bar{b}(K)$ and eliminate the crisis region altogether. Suppose the government issues and redeems $B_N$ bonds every period, sold at $\beta^N$. Then

$$B_N = \frac{1 - \beta}{1 - \beta^N} B$$

Since borrowing is lower every period, so is the probability of a default.

- This argument relies on the maturity structure of prevailing, and not new debt

- Crises can arise if large enough repayments are upcoming.
Sunspot equilibria arise due to coordination failure on the part of lenders.

Very similar to Diamond-Dybvig!

A “lender of last resort” has the potential to eliminate such equilibria just like in DD

Conesa and Kehoe (2011): govts may optimally *gamble for redemption* and expose themselves to self-fulfilling crises, in which case the above argument does not work.