Self-Fulfilling Debt Crises

Harold Cole and Timothy Kehoe (RES, 2000)

Presented by Miguel de Faria e Castro

New York University
Sargent Reading Group

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Mexican Crisis: 1994-1995

- Fear of government default led to the government being unable to issue new debt...
- ...in turn confirming the fear of default! (until the US intervened)

Mexican *fiscal fundamentals* were stable and comparable to those of healthy governments.

Key Feature: average maturity of Mexican debt had become very short.

Self-Fulfilling Debt Crises

A Simple Example

Risk-neutral investors, deep pockets, no constraints

$$\underbrace{q}_{\text{price of bond}} = \beta \underbrace{\mathbb{E}\mathcal{H}}_{\text{expected haircut}}$$

Government budget constraints:

$$t = 0: \quad qB = D_0$$
$$t = 1: \quad 0 = D_1 + \mathcal{H}B$$

Then

$$\mathcal{H} = -q\frac{D_1}{D_0}$$

So $\mathcal{H}$ depends on its own expected value!

This Paper

Main questions:

1. How can debt crises such as the Mexican one arise in the context of a rational expectations DSGE?
2. What should optimal policy responses be, from a benevolent government under the threat of such crises?

Preview of the results:

1. **Sunspot equilibria**: aggregate outcomes determined by the beliefs of market participants for certain regions of the fundamentals;
2. Governments have incentives to reduce debt and abandon the "crisis zone" since that generates booms and reductions in yields;
3. Increasing debt maturity shrinks the crisis zone (Mexican problem);
4. Usual credibility enhancement policy remedies may be counterproductive.

Environment

- Time is discrete, infinite horizon;
- Single good, consumed or saved as capital;
- Agents:
  1. Consumers (eat, save, produce);
  2. International bankers (eat, buy government debt);
  3. Government (taxes, spends, issues debt, may default).

Timing in each period:

1. sunspot variable ζ_t is realized; the aggregate state of the economy is
$$s_t = (B_t, K_t, a_{t-1}, \zeta_t)$$
2. government chooses B_{t+1}, taking $q(s_t, B_{t+1})$ as given
3. bankers choose b_{t+1}, taking q_t as given
4. government chooses how much to consume g_t and whether to default or not $z_t \in \{0, 1\}$
5. households choose c_t, k_{t+1}, taking a_t as given

Households and Bankers

Households solve:

$$V_c(k,s,B',g,z) = \max_{c,k'} \left\{ c + v(g) + \beta \mathbb{E} V_c(k',s',B'(s'),g',z') \right\}$$

$$\text{s.t.}$$

$$c + k' \leq (1-\theta) a(s,z) f(k)$$

$$c, k' \geq 0$$

$$s' = [B', K'(s,B',g,z), a(s,z), \zeta']$$

$$g' = g[s', B(s'), q(s', B'(s'))]$$

$$z' = z[s', B'(s'), q(s', B'(s'))]$$

Bankers solve:

$$V_b(b,s,B') = \max_{b'} \left\{ \bar{x} + z[s,B',q(s,B')]b - q(s,B')b' + \beta \mathbb{E} V_b(b',s',B'(s')) \right\}$$

$$\text{s.t.}$$

$$q(s,B')b' \leq \bar{x}$$

$$b' \geq -A$$

$$s' = [B', K'(s,B',g,z), a(s,z), \zeta']$$

Government

First, chooses how much debt to issue

$$V_g(s) = \max_{B'} \left\{ c(K, s, B', g, z) + v(g) + \beta \mathbb{E} V_g(s') \right\}$$

$$\begin{aligned} &\text{s.t.} \\ &g = g[s, B', q(s, B')] \\ &z = z[s, B', q(s, B')] \\ &s' = [B', K'(s, B', g, z), a(s, z), \zeta'] \end{aligned}$$

Then, how much to consume and whether to default or not

$$\max_{g \geq 0, z \in \{0,1\}} \left\{ c(K, s, B', g, z) + v(g) + \beta \mathbb{E} V_g(s') \right\}$$

$$\begin{aligned} &\text{s.t.} \\ &g + zB \leq \theta a(s, z) f(K) + qB' \\ &s' = [B', K'(s, B', g, z), a(s, z), \zeta'] \end{aligned}$$

Equilibrium

A recursive equilibrium is:

- Value functions: V_c, V_b, V_g
- Policy functions: c, k', b', and B', g, z
- Price function q
- Law of motion for capital K'

such that:

- Given B', g, z, V_c solves the household problem and (c, k') are the optimizing policies
- Given B', q, z, V_b solves the banker's problem and B' chosen by the government solves the problem when $b = B$
- Given q, c, K', g, z, V_g solves the government's (first) problem, and B' is the optimizing policy. Also, given c, K', V_g, B', (g, z) solve the government's second problem.
- $B'(s) \in b'(B, s, B')$
- $K'(s, B', g, z) = k'(K, s, B', g, z)$

The Crisis Zone

Self-fulfilling crises arise when there are two possible equilibrium outcomes.

- Possible for certain values of the fundamentals (B, K);
- Sunspot variable determines which outcome ensues.

Let $\pi \in [0, 1]$ parametrize the probability of a crisis.

Define the **crisis zone** as

$$\bar{b}(K) \leq B \leq \bar{B}(K, \pi)$$

- If $B \leq \bar{b}(K)$, no crisis occurs $\forall \zeta$;
- If $B \geq \bar{B}(K, \pi)$, a crisis always occurs $\forall \zeta$.
- If $B \in CZ$, bankers predict default if $\zeta \leq \pi$ and repayment if $\zeta > \pi$.

Private Agents

- **Bankers** - Depending on their beliefs, can offer to buy up to $\bar{x}$ in value of govt debt at prices

$$q(B') = \begin{cases} \beta & \text{if } B' \leq \bar{b}(k^n) \text{ and } z = 1 \\ \beta(1-\pi) & \text{if } \bar{b}(k^n) \leq B' \leq \bar{B}(k^\pi, \pi) \text{ and } z = 1 \\ 0 & \text{otherwise} \end{cases}$$

- **Households** - Due to risk-neutrality, problem is easy to solve

$$K'(B') = \begin{cases} k^n & \text{if } B' \leq \bar{b}(k^n) \text{ and } z = 1 \\ k^\pi & \text{if } \bar{b}(k^n) \leq B' \leq \bar{B}(k^\pi, \pi) \text{ and } z = 1 \\ k^d & \text{otherwise} \end{cases}$$

where $k^n > k^\pi > k^d$.

Government Incentives

After B' is sold at a positive price, there is no default iff

$$\textbf{Participation Constraint}: \; V_g^n(s, B', q) \geq V_g^d(s, B', q)$$

For $q = \beta(1 - \pi)$, this constraint determines $\bar{B}(K, \pi)$.

For a crisis to be possible (and *credible*), the government must default whenever it is unable to sell debt at a positive price

$$\textbf{No-Lending Condition}: \; V_g^d(s, 0, 0) > V_g^n(s, 0, 0)$$

This determines $\bar{b}(K)$.

Whenever the above inequalities are satisfied, a crisis zone exists.

Policy outside the Crisis Zone

If $B_t \leq \bar{b}(k^n)$, can show that

$$g_t = g_{t+1}$$
$$B_t = B_{t+1}$$

as well as $K' = k^n$ and $q = \beta$ for all t.
No Crisis Zone is an Absorbing State

If $B_t > \bar{B}(K, \pi)$, the PC is violated and the government defaults.

Policy in the Crisis Zone

What if $B_0 \in [\bar{b}(K_0), \bar{B}(K_0, \pi)]$?

1. Default immediately;
2. Never run the debt down (**stationary policy**);
3. Run the debt down in $T < \infty$ periods;

How does this option work? The government will never run the debt below $\bar{b}(k^n) > \bar{b}(k^\pi)$, so:

1. For $t \in \{0, \ldots, T-2\}$, spending constant at $g^T(B_0)$ and consumption at c^π; probability of a crisis equal to π
2. At $T-1$, debt set to $B_T = \bar{b}(k^n)$, consumption equal to c^n if no crisis occurs;
3. From T onwards, both government spending and investment increase and the economy leaves the crisis zone permanently.

Characterizing the Zones

General characterization is difficult since one must consider all possible (K_0, B_0) combinations and government policies will generically not be stationary in the crisis zone.

Can show that $\exists B^s(\pi)$ for which debt is stationary in the crisis zone (PC in equality):

1. If $B_0 > B^s(\pi)$, PC binds and debt is immediately reduced;
2. If $B_0 < B^s(\pi)$, debt runs down in $T(B_0) < \infty$.

Sunspots and Self-Fulfilling Crises

Proposition

For any K_0 and $B_0 + \theta f(K_0) \leq B^s(\pi) + \theta f(k^\pi)$, let V_g^T denote the value of reducing the debt to $\bar{b}(k^n)$ in T periods. Then, a $T \in \{1, 2, \ldots, \infty\}$ that maximizes V_g^T exists and

1. *If $K_0 \geq k^\pi$, as B_0 increases, $T(B_0)$ passes critical points where it increases by one period.*
2. *If $K_0 < k^\pi$, the debt may increase in the first period but afterwards follows the characterization in 1.*

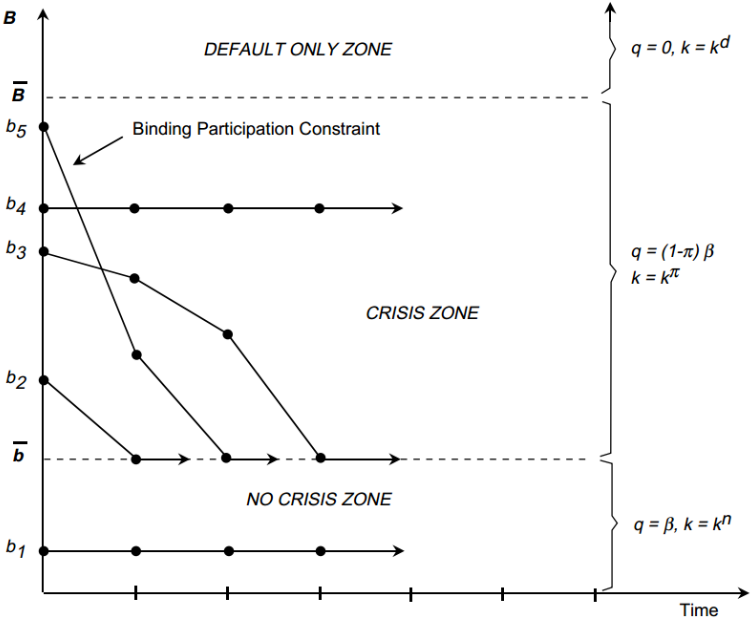
Intuition for 2: If B_0 is very high, the PC binds in period 0. The government either defaults or smooths g_t and sets $B_1 < \bar{B}$.

Main Result

For any probability $\pi > 0$ for which there exists a nonempty crisis zone $\bar{b}(k^n) < B \leq \bar{B}(k^\pi, \pi)$, there exists an equilibrium in which:

1. If $\bar{b}(K_0) \leq B_0 \leq \bar{B}(K_0, \pi)$, then a crisis occurs with probability π in the first period and every subsequent period in which $B > \bar{b}(k^n)$.
   - If $B_0 + \theta f(K_0) \leq B^s(\pi) + \theta f(k^\pi)$, optimal government policy involves running down the debt to $\bar{b}(k^n)$ in $T(B_0)$ periods.
   - If $B_0 + \theta f(K_0) > B^s(\pi) + \theta f(k^\pi)$, the government starts running down the debt in at most two periods.
2. If $B_0 \leq \bar{b}(k^n)$,
   - If $K_0 \geq k^n$, the economy is stationary in the no-crisis zone.
   - If $K_0 < k^n$ and $B_1 \leq \bar{b}(k^n)$, same as above.
   - If $K_0 < k^n$ and $B_1 > \bar{b}(k^n)$, everything proceeds as in 1.
3. If $B_0 > \bar{B}(K_0, \pi)$, the only outcome is default.

Debt Trajectories



Some Results

- General results depend on the initial level of capital.
- For B close to $\bar{b}(k^n)$, the government may be able to raise more revenue by selling more debt as

$$\beta(1-\pi)B < \beta\bar{b}(k^n)$$

  for π large enough.
- As $\pi \to 1$, this region increases and eventually encompasses the entire crisis zone: transition out of the crisis zone occurs within one period for any (B_0, K_0).

Extensions

- **Domestically Initiated Crises** - Crises may also be triggered by self-fulfilling fears of domestic investors: by setting $k^d < k^n$, they reduce tax revenues and may trigger default.

- **Temporary Cost of Default** - If $\alpha < 1$ for a finite horizon and the government may regain access to international markets, the set of equilibria expands. Sunspot equilibria in which the probability of a crisis depends on the history of past defaults become possible.

Policy Implications

- **Credibility** - Suppose the government sets $\downarrow \alpha$ as a (limited) commitment device. This "relaxes" the crisis region $\uparrow \bar{b}(K), \uparrow \bar{B}(K, \pi)$, but does not eliminate it, potentially worsening the consequences of a crisis.
- **Maturity** - By lengthening the maturity structure, the government can increase $\bar{b}(K)$ and eliminate the crisis region altogether.

  Suppose the government issues and redeems B_N bonds every period, sold at β^N. Then

  $$B_N = \frac{1-\beta}{1-\beta^N} B$$

  Since borrowing is lower every period, so is the probability of a default.
- This argument relies on the maturity structure of prevailing, and not new debt
- Crises can arise if large enough repayments are upcoming.

Policy Implications Cont'd

- Sunspot equilibria arise due to coordination failure on the part of lenders.
- Very similar to Diamond-Dybvig!
- A "lender of last resort" has the potential to eliminate such equilibria just like in DD
- Conesa and Kehoe (2011): govts may optimally *gamble for redemption* and expose themselves to self-fulfilling crises, in which case the above argument does not work.