The Macroeconomic Implications of Rising Wage Inequality in the United States
Heathcote, Storesletten, Violante (JPE, 2010)

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Motivation

• **Observation:** Wage inequality has changed along several dimensions since the 1960s
  1. Residual wage inequality has increased,
  2. College premium has increased,
  3. Gender premium has decreased.

• **Question 1:** Given these changes, can one account for the observed dynamics in
  1. Hours (women increase hours relative to men, variance of hours, correlation with wages)?
  2. Household earnings?
  3. Household consumption?

• **Question 2:** Are households under the observed wage dynamics ex ante better off than if the wage structure of 1965 had prevailed forever?
Approach

Idea:

- Set up neoclassical growth model with incomplete markets and overlapping generations, and target the observed wage dynamics. Driving forces are
  1. Residual wage inequality: time-varying variance of permanent and transitory shock to income.
  2. College premium: skill-biased labor demand shift.

- Then see what output agents generate. Check whether output looks similar to the data (target output).
**Inputs (targeted in calibration)**

(A) Variance of Log Male Wages

(B) Male College Wage Premium

(C) Fraction of College Graduates (age 25–29)

(D) Female–Male Wage Ratio
Output (hold model up against this)
Demographics

- Each period a cohort of measure one of both males and females is born.
- Timeline:
  1. Individual decides whether to pursue education.
  2. Males and females are matched into households (probabilities depend on education status).
  3. Household then jointly decides every period about labor supply by each member, joint consumption and joint savings (in riskless bond).
  4. Both household members retire at age $j^R$ and receive retirement benefits.
- Survival stochastic, but terminal age $J$. 
Education and matching

Education

• Cost of college drawn from some gender- and cohort-specific distribution $F^g_t(\kappa)$.

• Let $M^g_t(e)$ be the expected value of entering the subsequent matching stage with education level $e$. Then the optimal education choice is

$$e^g_t(\kappa) = \begin{cases} h & \text{if } M^g_t(h) - \kappa \geq M^g_t(l), \\ l & \text{otherwise.} \end{cases}$$

Matching

• $\pi^m_t(e^f, e^m)$ is the probability that a man with education level $e^m$ is assigned to a woman with education $e^f$.

• Since the share of college vs. high-school educated individuals varies over cohorts, the matching probabilities must be time-varying as well in order to reach a target level of assortative matching.
If an individual works for one hour, she provides a certain number of efficiency units of labor, based on experience and idiosyncratic labor productivity shocks.

\[
\text{wage} = p_t^{g,e} \times \exp[L(j) + y_t],
\]

where

\[
y_t = \eta_t + \nu_t,
\]

\[
\eta_t = \rho \eta_{t-1} + \omega_t,
\]

is the stochastic process for the idiosyncratic productivity. Shocks are Gaussian with variances \{\lambda^\eta, \lambda^\nu, \lambda^\omega\}. Notice: both spouses have their own idiosyncratic productivity.
Problem of a working household

\[
\nabla_t(e^m, e^f, j, a_t, y^m_t, y^f_t) = \max_{c_t, a_{t+1}, n^m_t, n^f_t} \left( u(c_t, n^m_t, n^f_t) \right.
\]

\[
+ \beta \zeta^j \mathbb{E}_t[\nabla_t(e^m, e^f, j + 1, a_{t+1}, y^m_{t+1}, y^f_{t+1})]
\]

subject to

\[
c_t + \zeta^j a_{t+1} = [1 + (1 - \tau^a)r]a_t + (1 - \tau^n)[p^m_t, e^m_t \epsilon(j, y^m_t)n^m_t + p^f_t, e^f_t \epsilon(j, y^f_t)n^f_t]
\]

\[
a_{t+1} \geq a, c_t \geq 0; n^m_t, n^f_t \in [0, 1]
\]

- Will use \( u(c, n^m, n^f) = \frac{c^{1-\gamma}}{1-\gamma} + \psi \frac{(1-n^m)^{1-\sigma}}{1-\sigma} + \psi \frac{(1-n^f)^{1-\sigma}}{1-\sigma} \).

- Notation: \( \mu_t \) is the distribution over the state space (four education-gender pairs, age, assets, persistent and transitory income component). Call \( \mu_\ast \) the initial stationary distribution.
Production

- Representative firm using CD technology with inputs capital and aggregate labor, $Z_t K_t^\alpha H_t^{1-\alpha}$, where

$$H_t = \{\lambda_t^S [\lambda_t^G H_t^{h,f}]^{\theta-1} + (1-\lambda_t^S)\} \frac{\theta-1}{\theta} \}^{\theta-1}$$

- Conditional on education, male and female labor are perfect substitutes.
- Skill- and gender-biased demand shifts will work through $\lambda_t^S$ and $\lambda_t^G$. 
Definition of equilibrium

Given $\mu_*$, and sequences $\{\lambda_t\}$ and $\{Z_t, F^g_t\}$ a competitive equilibrium is a sequence of discounted values at each stage, decision rules for education, consumption, hours worked, and savings; firm choices $\{H^g_t, e_t, K_t\}$; prices $\{p^g_t, e_t\}$; government expenditures; cohort- and gender specific college enrollment rates; and a measure of households $\{\mu_t\}$, such that for all $t$, the following are satisfied:

1. Education decision solves the individual’s problem.
2. Decision rules for hours, consumption and savings solve the household’s problem.
3. Labor are allocated optimally, i.e.,

\[
\begin{align*}
    p^{m,h}_t &= \Omega^h_t (1 - \lambda^G_t) \lambda^S_t, \\
    p^{m,l}_t &= \Omega^l_t (1 - \lambda^G_t)(1 - \lambda^S_t), \\
    p^{f,h}_t &= \Omega^h_t \lambda^G_t \lambda^S_t, \\
    p^{m,h}_t &= \Omega^l_t \lambda^G_t (1 - \lambda^S_t), \\
    \Omega^e_t &= (1 - \alpha) Z_t K^\alpha_t H^{(1/\theta)-\alpha}_t H^{*,e}_t + (1 - \lambda^G_t) H^{m,e}_t -^{1/\theta}_t,
\end{align*}
\]
and capital is allocated optimally, i.e.,

\[ r = \alpha Z_t \left( \frac{H_t}{K_t} \right)^{1-\alpha} - \delta. \]

4. All four labor markets clear.
5. Domestic good market clears, i.e.

\[ C_t + K_{t+1} - (1 - \delta)K_t + G_t + NX_t = Z_t K_t^\alpha H_t^{1-\alpha} \]

6. World asset market clears.
7. Government budget is balanced.
8. Sequences of measures \( \{\mu_t\} \) is consistent with household decision rules.
Model period 1 year. Individuals start life at age 25, work until age 60.

Suppose it is 1965 and we are in steady state, i.e. prices are, always have been, and are expected to always remain \( \{ p_{1965}^{g,e} \} \), parameters \( \{ \lambda_{1965} \} \) and \( \{ Z_{1965}, F_{1965}^g \} \).

Now, we reveal to agents the new path for prices and parameters for \( t = 1966, \ldots, \infty \), with a new steady state to be reached in 2021.

Prices and parameters will vary until 2021, but will be constant thereafter.
## Parametrization

### TABLE 1
**Summary of Parameterization**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Moment to Match</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters set externally:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>${\xi^j}$</td>
<td>Age-specific survival rates (U.S. Life Tables)</td>
<td>See text</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Micro estimates of relative risk aversion</td>
<td>1.50</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>Intrafamily correlation of education at ages 25–35</td>
<td>.517</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of output (NIPA)</td>
<td>.33</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate (NIPA)</td>
<td>.06</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of substitution between college and high school graduates</td>
<td>1.43</td>
</tr>
<tr>
<td>$r$</td>
<td>Before-tax risk-free interest rate</td>
<td>.05</td>
</tr>
<tr>
<td>$\tau^n, \tau^a$</td>
<td>Labor income and capital income tax rates</td>
<td>.27, .40</td>
</tr>
<tr>
<td>$L(j)$</td>
<td>Male hourly wage life cycle profile</td>
<td></td>
</tr>
<tr>
<td>${\lambda_i^e, \lambda_i^e, \lambda^u, \rho}$</td>
<td>Male hourly wage residuals dynamics</td>
<td>See fig. 3</td>
</tr>
</tbody>
</table>
## Parametrization

Parameters calibrated internally:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Ratio of average wealth (for poorest 99%) to average labor income</td>
<td>.969</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Average household market hours</td>
<td>.335</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Ratio of average male to female market hours</td>
<td>3.0</td>
</tr>
<tr>
<td>$b$</td>
<td>Redistribution (of lifetime earnings) through U.S. pension system</td>
<td>.336</td>
</tr>
<tr>
<td>$a$</td>
<td>15.5% of households with zero or negative wealth</td>
<td>$-0.20$</td>
</tr>
<tr>
<td>${\lambda_{i}^m}$</td>
<td>Ratio of average male college to high school wages</td>
<td>See fig. 3</td>
</tr>
<tr>
<td>${\lambda_{i}^f}$</td>
<td>Ratio of average male to female wages, full-time workers</td>
<td>See fig. 3</td>
</tr>
<tr>
<td>${Z_i}$</td>
<td>Average posttax earnings equal to one, with no behavioral response</td>
<td>See text</td>
</tr>
<tr>
<td>$\bar{k}_m^m$, $\nu_k^m$</td>
<td>Male college enrollment in initial and final steady state</td>
<td>2.96, .88</td>
</tr>
<tr>
<td>$\bar{k}_f^f$, $\nu_k^f$</td>
<td>Female college enrollment in initial and final steady state</td>
<td>2.22, .31</td>
</tr>
<tr>
<td>${\bar{k}_i^m$, $\bar{k}_i^f}$</td>
<td>Male and female college enrollment during the transition</td>
<td>See text</td>
</tr>
</tbody>
</table>
Calibration strategy and challenges

- Start with determining the initial and final steady states. Guess prices, derive decision rules and simulate the economy. Check whether target values of wage dispersion, wage premiums, enrollment rates, and others are matched. Update guesses and iterate. Obtain stationary distribution.

- Once we have found the steady states, we can fill in the transitional dynamics by guessing sequences for prices and parameters.

- For $t \geq 2000$, skill- and gender-premia are constant at the year 2000 level, but $\lambda_t^G$ and $\lambda_t^S$ are time-varying until year 2021, because skill composition of the economy is still changing. Only thereafter, prices and parameters are constant.

- Set $Z_t$ such that labor productivity does not change based on labor demand shifts alone, but through deliberate decisions about education and hours.
Hours

(A) Female / Male Hours Worked

(B) Variance of Log Male Hours

(C) Variance of Log Female Hours

(D) Decomposition

(E) Decomposition

(F) Decomposition
Wage-hours correlation

(A) Male Wage–Hour Correlation

(B) Female Wage–Hour Correlation

(C) Decomposition

(D) Decomposition
Earnings and consumption inequality

(A) Variance of Log Household Earnings

(B) Variance of Log Household Consumption

(C) Decomposition

(D) Decomposition
Welfare

Welfare criterion

\[ 2\mathbb{E}_t \left\{ \sum_{j=0}^{J-1} \beta^j \bar{\zeta}_j u(c_{t+j}, n^m_{t+j}, n^f_{t+j})|e^m, e^f \right\} - \sum_{g \in \{m,f\}} \mathbb{I}\{e^g = h\} \mathbb{E}_* [\kappa | \kappa \leq \hat{\kappa}_g^*] \]

\[ = 2\mathbb{E}_* \left\{ \sum_{j=0}^{J-1} \beta^j \bar{\zeta}_j u((1 + \phi_t)c_{*j}, n^m_{*j}, n^f_{*j})|e^m, e^f \right\} - \sum_{g \in \{m,f\}} \mathbb{I}\{e^g = h\} \mathbb{E}_* [\kappa | \kappa \leq \hat{\kappa}_g^*] \]

- Myopic version: At each \( t \) all households believe that wage dispersion and wage premia will remain at the current level. Again, need to guess sequences for parameters and prices, then solve households problem backwards, taking into account its myopic expectations.
Welfare under perfect foresight
Welfare under myopic beliefs