Inequality and Social Discounting

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Outline

1. Motivation: environment, the immiseration result
2. Solving: a relaxed problem, a recursive formulation
3. Results: a mean-reverting process, a steady-state distribution without misery

Environment

- **Endowment economy** with constant resources $e\ \forall t$
- Continuum of **consumers** (measure 1):
  - Preferences: $\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \theta_t U(c_t)$
  - Idiosyncratic i.i.d. taste shock $\theta_t \in \Theta = \{\theta_1, ..., \theta_n\}$ w.p. $\pi_t(\theta^t)$
  - Each consumer enters the economy with a number v his entitlement to expected discounted utility (initial distribution ψ)
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Strategies:

- σ a reporting strategy: $\sigma = \{\sigma_t(\theta^t)\}_{t=0}^{\infty}$, $\sigma_t : \Theta^{t+1} \to \Theta$.
- σ^* a truthful reporting strategy: $\sigma^*(\theta^t) = \theta_t$ $\forall t$ $\forall \theta^t \in \Theta^{t+1}$

Allocations

Definition

An allocation is a sequence $\{u_t^v\}_{t=0}^{\infty}$ $\forall v$, where $u_t^v(\theta^t) \equiv U(c_t(\theta^t, v))$.

Promise-keeping constraint $\forall v$:

$$v = \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \beta^t \pi_t(\theta^t)\theta_t u_t^v(\theta^t) \tag{PK}$$

Incentive-compatibility constraint $\forall v$:

$$\sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \beta^t \pi_t(\theta^t)\theta_t[u_t^v(\theta^t) - u_t^v(\sigma^t(\theta^t)] \geq 0 \ \forall \sigma \tag{IC}$$

Resource constraint $\forall t$ ($C \equiv U^{-1}$):

$$\int \sum_{\theta^t \in \Theta^{t+1}} \pi_t(\theta^t) C(u_t^v(\theta^t)) d\psi(v) \leq e \tag{RC}$$

Atkeson & Lucas (1992)

Planner Efficiency Problem given ψ:

$$e^*(\psi) = \min_{\{u^v\}} e \quad \text{s.t. } (IC), (PK) \text{ and } (RC) \text{ hold}$$

An immiseration result: *Welfare and consumption inequality rise steadily without bound, with everyone converging to absolute misery.*

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Intuition:

- Rewards and punishments, required for incentives...
- ... are best delivered permanently, to smooth consumption over time

Consumption process inherits a random-walk component:

- Cross-sectional inequality grows unboundedly...
- ... but aggregate endowment is constant

Everyone's consumption has to converge to zero.

Farhi & Werning (2007)

A minimal departure from A&L framework: $\hat{\beta} > \beta$

- An *intergenerational model*, where the planner places a **positive** and vanishing **Pareto weight** on the welfare of *future generations*

Farhi & Werning (2007)

A minimal departure from A&L framework: $\hat{\beta} > \beta$

- An *intergenerational model*, where the planner places a **positive** and vanishing **Pareto weight** on the welfare of *future generations*
  - Suppose a two-period model with two generations of one-period agents.
    - In $t = 1$ the child consumes c_1: $v_1 = U(c_1)$
    - In $t = 0$, the parent consumes c_0 and cares for the welfare of its child: $v_0 = U(c_0) + \beta v_1$
  - A&L: $W \equiv v_0$
  - A welfare criterion that weighs both agents is $W \equiv v_0 + \alpha v_1 = U(c_0) + \hat{\beta} U(c_1)$ with $\hat{\beta} = \alpha + \beta$

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  - A welfare criterion that weighs both agents is $W \equiv v_0 + \alpha v_1 = U(c_0) + \hat{\beta} U(c_1)$ with $\hat{\beta} = \alpha + \beta$
- Generates a trade-off between efficiency and insurance for future generations: a mean-reverting tendency in consumption

A Social Planning Problem (SPP)

The Social Planning Problem is:

$$S(\psi, e) \equiv \sup_{\{u_t^v\}} \int \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \hat{\beta}^t \pi_t(\theta^t)[\theta_t u_t^v(\theta^t)] d\psi(v)$$

s.t.

$$\sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \beta^t \pi_t(\theta^t)\theta_t u_t^v(\theta^t) = v \quad \forall v \qquad \text{(PK)}$$

$$\sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \beta^t \pi_t(\theta^t)\theta_t [u_t^v(\theta^t) - u_t^v(\sigma^t(\theta^t))] \geq 0 \quad \forall \sigma \quad \forall v \qquad \text{(IC)}$$

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+ Cases where $e > e^*(\psi)$.

+ We seek **steady-state distributions** $\psi^* = \Psi\psi^*$ where aggregate consumption is strictly positive.

A Relaxed Social Planning Problem

The Relaxed SPP is:

$$\hat{S}(\psi, e) \equiv \sup_{\{u_t^v\}} \int \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \hat{\beta}^t \pi_t(\theta^t)[\theta_t u_t^v(\theta^t)] d\psi(v)$$

s.t. (PK), (IC) and

$$\int \sum_{t=0}^{\infty} Q_t \sum_{\theta^t \in \Theta^{t+1}} \pi_t(\theta^t) C(u_t^v(\theta^t)) d\psi(v) \leq e \sum_{t=0}^{\infty} Q_t$$

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In **steady-state**, $Q_t = \hat{\beta}^t$ and the two resources constraints are equivalent.

$$\int \sum_{t=0}^{\infty} \hat{\beta}^t \sum_{\theta^t \in \Theta^{t+1}} \pi_t(\theta^t) C(u_t^v(\theta^t)) d\psi(v) \leq e \sum_{t=0}^{\infty} \hat{\beta}^t \qquad (\eta)$$

A Recursive Component Planning Problem (CPP)

We can form the Lagrangian of the Relaxed SPP: $\mathcal{L} \equiv \int \mathcal{L}^v d\psi(v)$ s.t. (PK) and (IC) where

$$\mathcal{L}^v \equiv \sum_{t=0}^{\infty} \sum_{\theta^t \in \Theta^{t+1}} \hat{\beta}^t \pi_t(\theta^t)[\theta_t u_t^v(\theta^t) - \eta C(u_t^v(\theta^t))]$$

Component planning problems (pointwise optimization):

$$k(v) \equiv \sup_{u_t^v} \mathcal{L}^v \text{ s.t. } (PK)_v, \ (IC)_v$$

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Theorem

The value function of the CPP $k(v)$ is continuous, concave and satisfies the Bellman equation

$$k(v) = \max_{\{u(\theta), w(\theta)\}} \mathbb{E}[\theta u(\theta) - \eta C(u(\theta)) + \hat{\beta} k(w(\theta))] \text{ such that}$$

$$v = \mathbb{E}[\theta u(\theta) + \beta w(\theta)]$$
$$\theta u(\theta) + \beta w(\theta) \geq \theta u(\theta') + \beta w(\theta') \ \forall \theta, \theta' \in \Theta$$

Characterization of the policy functions

Let $g^u(\theta, v) = u^*(\theta, v)$ and $g^w(\theta, v) = w^*(\theta, v)$ the continuous **policy functions**.

An **allocation** $\{u_t^v\}$ with initial entitlement v is **generated** from (g^u, g^w) if $u_t(\theta^t) = g^u(\theta_t, v_t(\theta^{t-1}))$, $v_{t+1}(\theta^t) = g^w(\theta_t, v_t(\theta^{t-1}))$ and $v_0 = v$.

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Theorem

- *For any (v, η), if an allocation $\{u_t\}$ attains the maximum of the CPP, then it is generated by (g^u, g^w).*
- *Conversely, if an allocation $\{u_t\}$ generated by (g^u, g^w) is such that $\forall\sigma$*

$$\limsup_{t\to\infty} \mathbb{E}_{-1}\beta^t v_t(\sigma^{t-1}(\theta^{t-1}) \geq 0$$

then it attains the maximum of the CPP.

A Mean-Reverting Process

The **recursive problem** to solve is:

$$k(v) = \max_{\{u(\theta),w(\theta)\}} \mathbb{E}[\theta u(\theta) - \eta C(u(\theta)) + \hat{\beta} k(w(\theta))] \quad \text{s.t.}$$

$$v = \mathbb{E}[\theta u(\theta) + \beta w(\theta)]$$

$$\theta u(\theta) + \beta w(\theta) \geq \theta u(\theta') + \beta w(\theta') \ \forall \theta, \theta' \in \Theta$$

Proposition

Let $\underline{v} \equiv U(0)/(1-\beta)$ *and* $\bar{v} \equiv U(+\infty)/(1-\beta)$. $k(v)$ *is differentiable and strictly concave, with* $\lim_{v \to \bar{v}} k'(v) = -\infty$ *and* $\lim_{v \to \underline{v}} k'(v) \geq 1$.

The Markov process $\{k'(v_t)\}$ regresses towards zero:

$$\mathbb{E}_{t-1}[k'(v_{t+1})] = \frac{\beta}{\hat{\beta}} k'(v_t)$$

Existence of a Steady-State Distribution

Proposition

For $k'(v) \leq 1$, $\exists\ \underline{\gamma}, \bar{\gamma} \in \mathbb{R}$ *with* $\underline{\gamma} \leq \frac{\beta}{\hat{\beta}} \leq \bar{\gamma}$ *and* $\underline{\gamma}$, $\bar{\gamma} \to \frac{\beta}{\hat{\beta}}$ *as* $\frac{\bar{\theta}}{\underline{\theta}} \to 1$

$$\underline{\gamma}[1 - k'(v)] + (1 - \frac{\beta}{\hat{\beta}}) \leq 1 - k'(g^w(\theta, v)) \leq \bar{\gamma}[1 - k'(v)] + (1 - \frac{\beta}{\hat{\beta}})$$

Moreover, $C(g^u(\theta, v))$ *is zero if and only if* $k'(v) \geq 1$.

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Moreover, $C(g^u(\theta, v))$ *is zero if and only if* $k'(v) \geq 1$*.*

Proposition

The Markov process $\{v_t\}$ *implied by* g^w *has an* **invariant distribution** ψ^* *with no mass at misery* $\psi^*(\underline{v}) = 0$ *and* $\int k'(v)d\psi^*(v) = 0$ *if any of the following conditions holds: utility is unbounded below, or bounded above, or* $\bar{\gamma} < 1$ *or* $\underline{\gamma} > 0$*.*

Conclusion

A **social discount factor** larger than the private discount factor:

- ... generates a trade-off efficiency vs. insurance
- ... generates a mean-reverting process for welfare and consumption

The immiseration result fails to hold.