Information Acquisition and Welfare

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Introduction

- This paper studies the effects of information acquisition in a "Quadratic-Gaussian" setup as introduced in Angeletos and Pavan (2007)
- Contribute to the debate on welfare implication of public information
  - previous studies ignored the impact of public information on private information production
- The authors:
  - characterize amount of private information acquired in the setup
  - relate inefficiency in information acquisition to inefficiency in equilibrium behavior
  - show how endogeneity of private information affects social value of public information
  - apply results to a few application of interest (e.g. Beauty Contest model of Morris and Shin, 2002)

Information Structure

- There is a continuum of agents, $i \in [0, 1]$
- The economy is characterized by an unknown parameter θ (economic fundamentals)
- All agents share the same prior regarding θ: $\theta \sim N\left(\theta_{-1}, p_\theta^{-1}\right)$
- Agents receive public and private signals regarding θ

$$
\begin{aligned}
y &= \theta + \varepsilon \quad , \quad \varepsilon \sim N\left(0, p_y^{-1}\right) \\
x_i &= \theta + \xi_i \quad , \quad \xi_i \sim N\left(0, p_x^{-1}\right)
\end{aligned}
$$

Public signal is a common knowledge among agents while private signal x_i is only observed by agent i

Private and Public Posteriors

- We define a public posterior as

$$\theta | y \sim N\left(z, p_z^{-1}\right)$$

where $z \equiv \frac{p_\theta \theta_{-1} + p_y y}{p_\theta + p_y}$ and $p_z \equiv p_\theta + p_y$

- Private posterior

$$\theta | y, x_i \sim N\left(\delta z + (1 - \delta) x_i, (p_x + p_z)^{-1}\right)$$

where

$$\delta = \frac{p_z}{p_z + p_x}$$

Preferences

- Agents have utility $U(k_i, K, \sigma_k, \theta)$ where $k_i \in \mathbb{R}$, $K = \int k_i di$ and $\sigma_k = \left(\int (k_i - K)^2\right)^{1/2} dk_i$
- Agents maximized their ex-ante utility:
  - they first choose precision of their private signal
  - they observe signals and make simultaneously their decisions
- Information is costly: cost of precision p_x is $C(p_x)$
  - $C'(p_x) > 0$, $C''(p_x) > 0$
  - $C(0) = 0$ and $C'(0) = 0$, $\lim_{p_x \to \infty} C'(p_x) = \infty$

Agent's Problem

- At $t = 0$ agent i chooses his precision of private signals to maximize
$$E\left[U\left(k_i, K, \sigma_k, \theta\right)\right] - C\left(p_{x_i}\right)$$
where expectation is taken over $\{x_i, y, \theta\}$
- At $t = 1$, after observing x_i and y, agent chooses action k_i to maximize
$$E\left[U\left(k_i, K, \sigma_k, \theta\right) | x_i, y\right]$$
where expectation is taken over θ
- In the complete information setup (θ known) there is unique equilibrium with

$$k_i = \kappa^*\left(\theta\right) = \kappa_0^* + \kappa_1^*\theta$$

with $\kappa_0^* = -\frac{U_k(0,0,0,0)}{U_{kk}+U_{kK}}$ and $\kappa_1^* = -\frac{U_{k\theta}}{U_{kk}+U_{kK}}$

Proposition

There exists unique symmetric equilibrium, where each agent chooses to acquire private information of precision p_x^* implicitly defined by

$$p_x^* = \sqrt{\frac{|U_{kk}| \left(\kappa_1^*\right)^2}{2C'\left(p_x^*\right)}} - \frac{p_z}{1-\alpha}$$

and take action $k^*(x_i, y) = \kappa_0^* + \kappa_1^*\left(\gamma^* z + (1-\gamma^*)\, x_i\right)$where

$$\gamma^* = \frac{\delta}{1-\alpha^*(1-\delta)} \text{ and } \alpha^* = \frac{|U_{kK}|}{Ukk}$$

- The amount of private information collected is decreasing in α^*, p_z and in the cost of information acquisition
- The substitutability between public and private information is increasing in the equilibrium degree of coordination: $\frac{\partial^2 p_x^*}{\partial \alpha^* \partial p_z} \leq 0$

Efficient Information Acquisition

- A social planner maximizes the ex-ante utility of a representative agent

$$\max_{p_x, k(x,y)} \int_{(\theta,y,x)} U(k, K, \sigma, \theta)\, dP(\theta, y, x; p_z, p_x) - C(p_x)$$

- Let $W(K, \sigma, \theta)$ denote welfare under a utilitarian aggregator, i.e. $W(K, \sigma, \theta) = \int U(k_i, K, \sigma, \theta)\, di$

Proposition

The efficient action is $k^{**}(x, y) = \kappa_0^{**} + \kappa_1^{**}(\gamma^{**} z + (1 - \gamma^{**}) x)$ *where*

$$\alpha^{**} = 1 - \frac{W_{KK}}{W_{\sigma\sigma}} \text{ and } \gamma^{**} = \frac{\delta}{1 - \alpha^{**}(1 - \delta)}$$

and the efficient collection of private information is

$$p_x^{**} = \sqrt{\frac{|W_{\sigma\sigma}| (\kappa_1^{**})^2}{2C'(p_x^{**})}} - \frac{p_z}{1 - \alpha^{**}}$$

Proposition

1. *Consider economies that are efficient in their use of information ($\kappa^* = \kappa^{**}$ and $\alpha^* = \alpha^{**}$). Then $p_x^* < p_x^{**}$ if and only if $U_{\sigma\sigma} < 0$ and $p_x^* > p_x^{**}$ if $U_{\sigma\sigma} > 0$*
2. *Consider economies that are efficient under complete information but inefficient in their use of information ($\kappa^* = \kappa^{**}$ and $\alpha^* \neq \alpha^{**}$). Then $p_x^* < p_x^{**}$ if and only if $\alpha > \alpha^{**}$ and $p_x^* > p_x^{**}$ if $\alpha^* < \alpha^{**}$*
3. *Consider economies that are inefficient under complete information but are efficient in their use of information ($\alpha^* = \alpha^{**}$ but $\kappa^* \neq \kappa^{**}$). Then $p_x^* < p_x^{**}$ if and only if $\kappa_1^* < \kappa_1^{**}$ and $p_x^* > p_x^{**}$ if $\kappa_1^* > \kappa_1^{**}$*

- The same conclusions hold if the planner can only choose precision of private information but could not affect the way society uses information. Denote the precision of private information chosen by planner in this case by $\widehat{p}_x^{**}$

Social Value of Public Information

- Let $w(p_x, p_z) = E[U(k_i, K, \sigma, \theta) | p_x, p_z] - C(p_x)$
- Social value of public information is defined as $\frac{\partial w(p_x, p_z)}{\partial p_z}$

Proposition

Recognizing endogeneity of public information reduces the social value of public information in economies where $\kappa_1^* \leq \kappa_1^{**}$, $\alpha^* \geq \alpha^{**}$ and $U_{\sigma\sigma} \leq 0$. It increases the social value of public information in economies where $\kappa_1^* \geq \kappa_1^{**}$, $\alpha^* \leq \alpha^{**}$, and $U_{\sigma\sigma} \geq 0$.

Proposition

Take any economy where the amount of private information collected in equilibrium is inefficiently low (i.e. $p_x^* < \widehat{p}_x^{**}$). There exists a critical threshold Δ such that if $\alpha^* - \alpha^{**} < \Delta$ then welfare always increases in the precision of public information

Beauty Contest Model:

- $$U\left(k_i, K, \sigma_k, \theta\right) = -\left(1-r\right)\left(k_i-\theta\right)^2 - r\left(L_i - \overline{L}\right)$$
  $$L_i = \int \left(k_i - k_j\right)^2 dj \;, \;\; \overline{L} = \int L_i di \text{ and } C\left(p_x\right) = \frac{p_x^{1+\eta}}{1+\eta}$$
- In this context $\alpha^{**} = 0 < r = \alpha^*$, $\kappa^* = \kappa^{**}$ and $U_{\sigma\sigma} > 0$
- It was shown by Morris and Shin(2002) that in this setting an increase in the precision of public information increases welfare iff
  $$p_z \geq \left(2r-1\right)\left(1-r\right)p_x$$
- Recognizing endogeneity of private information increases the range of parameters for which social value of public information is positive iff
  $$p_z \geq \left(2r - 1 - 2 - 2\frac{1-r}{\eta}\right)\left(1-r\right)p_x$$