Intermediary Leverage Cycles and Financial Stability

Tobias Adriand and Nina Boyarchenko

Discussion by Gaston Navarro

October 2, 2012

Motivation

- **financial intermediaries leverage is procyclical:** periods of asset value growth are associated wih leverage growth
  (Adrian & Shin (2010))
- Why is this puzzling?

$$\text{Leverage} = \frac{\text{Assets}}{\text{Assets} - \text{Liabilities}}$$

  If the portfolio is fixed $\Rightarrow\uparrow$ Assets $\rightarrow\downarrow$ Leverage
- Need a very reactive portfolio decision to have procyclical leverage. Many models with financial intermediation do not obtain this.
  (Brunnermeier & Sannikov (2012), He & Krishnamurthy (2012))

Motivation

This paper:

- introduces a new risk-based financial constraint that generates procyclical intermedary leverage
- developes a highly tractable framework

Mechanism: risk-based constraint

- two periods $t = 0, 1$
- risk-neutral investor with initial wealth is w_0
- two assets: capital k with price $p_{k,0}$, risk-free bond b with price 1
- assume $r_b = 0$, and $\frac{p_{k,1} - p_{k,0}}{p_{k,0}} = r_k \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu > 0$
- budget at $t = 0$: $w_0 = p_{k,0}k + b$
- return at $t = 1$:

$$\begin{aligned} w_1 &= p_{k,1}k + b \\ &= r_k p_{k,0} k + w_0 \end{aligned}$$

Mechanism: risk-based constraint

VaR constraint: $\{(b,k) : \mathbb{P}(w_1 \leq 0) \leq 1-\alpha\}$
Solution: $\max k : \mathbb{P}(r_k p_{k,0} k + w_0 \leq 0) = 1-\alpha$
Analogously:

$$\max \theta : \mathbb{P}(r_k \theta + 1 \leq 0) = 1-\alpha$$

where $\theta = \frac{p_{k,0} k}{w_0}$ is leverage.
Then, θ solves

$$\int_{-\infty}^{-1/\theta} r f_{\mu,\sigma}(r) dr = 1-\alpha$$

pdf show

Mechanism: risk-based constraint

In general: $\Uparrow \sigma \rightarrow \Downarrow \theta$

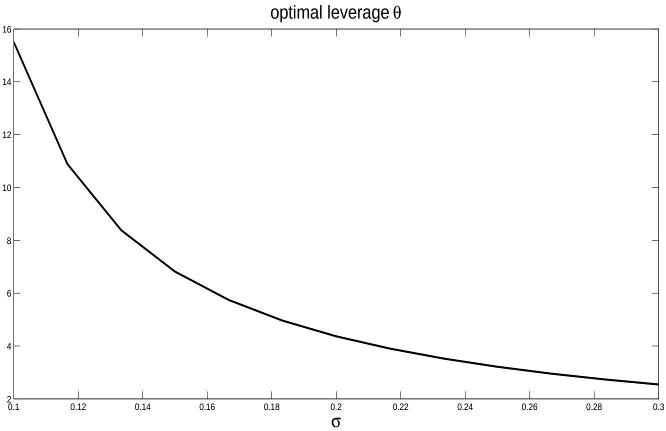

Figure: solution for θ

Model - Description

1. time is continuous and indexed by $t \geq 0$
2. two agents: financial intermediaries and households
3. production technology
4. productivity shock and household discount factor shock
5. three frictions:
   5.1 only financial intermediaries can produce new capital
   5.2 financial intermediaries may default on their debt
   5.3 financial intermediaries face a risk-based capital constraint

Model

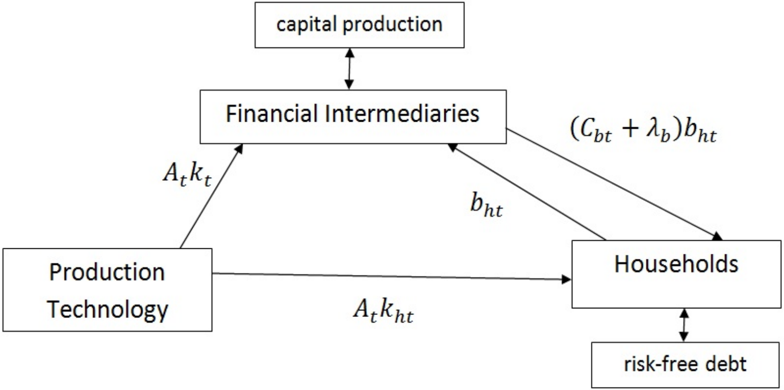

Figure: Model Structure

Model - Production

Output

$$Y_t = A_t K_t$$

where $A_t = e^{a_t}$ and

$$da_t = \bar{a}dt + \sigma_a dZ_{at}$$

where $\{Z_{at}\}_{t \geq 0}$ is a standard Brownian motion.

Model - Household

- capital and debt evolves as follows

$$dk_{ht} = -\lambda_k k_{ht} dt, \;\; db_{ht} = (\beta_t - \lambda_b) b_{ht} dt$$

- $p_{kt} A_t$ = price of capital, $p_{bt} A_t$ = price of debt
- Return on a unit of capital is

$$dR_{kt} = \underbrace{\frac{A_t k_{ht}}{k_{ht} p_{kt} A_t} dt}_{\text{dividend-price ratio}} + \underbrace{\frac{d(k_{ht} p_{kt} A_t)}{k_{ht} p_{kt} A_t}}_{\text{capital gains}}$$

- Return on a unit of intermediary debt is

$$dR_{bt} = \underbrace{\frac{(C_{bt} + \lambda_b) b_{ht}}{b_{ht} p_{bt} A_t} dt}_{\text{dividend-price ratio}} + \underbrace{\frac{d(b_{ht} p_{bt} A_t)}{b_{ht} p_{bt} A_t}}_{\text{capital gains}}$$

Model - Household

Let w_{ht} be household wealth. Then, his problem is

$$
\begin{aligned}
\max_{\{c_t, k_{ht}, b_{ht}\}} \quad & \mathbb{E}_0 \left[\int_0^{+\infty} e^{(-\xi_t + \rho_h t)} \log c_t dt \right] \\
\text{subject to} \quad & \\
dw_{ht} &= \left(w_{ht} - p_{kt} A_t k_{ht} - p_{bt} A_t b_{ht} \right) r_{ft} dt \\
&+ p_{kt} A_t k_{ht} dR_{kt} + p_{bt} A_t b_{ht} dR_{bt} - c_t dt \\
d\xi_t &= \sigma_\xi \rho_{\xi,a} dZ_{a,t} + \sigma_\xi \sqrt{1 - \rho_{\xi,a}^2} dZ_{\xi,t} \\
k_{ht} &\geq 0, \;\; b_{ht} \geq 0
\end{aligned}
$$

Model - Financial Intermediary

- capital and debt evolves as follows

$$dk_t = (\Phi(i_t) - \lambda_k)\, k_t dt, \;\; db_t = (\beta_t - \lambda_b) b_t dt$$

$\Phi(.)$ is investment technology: $\Phi(0) = 0$, $\Phi' > 0$, $\Phi'' < 0$

- total investment $= i_t A_t k_t$
- Return on a unit of capital is

$$\begin{aligned} dr_{kt} &= \frac{(1 - i_t) A_t k_t}{k_t p_{kt} A_t} dt + \frac{d(k_t p_{kt} A_t)}{k_t p_{kt} A_t} \\ &= dR_{kt} + \left(\Phi(i_t) - \frac{i_t}{p_{kt}} \right) dt \end{aligned}$$

- Return on a unit of debt is

$$\begin{aligned} dr_{bt} &= \frac{(C_{bt} + \lambda_b) b_t}{b_t p_{bt} A_t} dt + \frac{d(b_t p_{bt} A_t)}{b_t p_{bt} A_t} \\ &= dR_{bt} \end{aligned}$$

Model - Financial Intermediary

Let w_t be intermediary wealth. Then, his problem is

$$\max_{\{k_t, b_t, i_t\}} \quad \mathbb{E}_0\left[\int_0^{\tau_D} e^{-\rho t} w_t dt\right]$$

subject to

$$\begin{aligned} dw_t &= k_t p_{kt} A_t dr_{kt} - b_t p_{bt} A_t dr_{bt} \\ w_t &= \alpha \sqrt{\frac{1}{dt} \langle k_t d(p_{kt} A_t) \rangle^2} \end{aligned}$$

where

$$\tau_D = \inf_{t \geq 0} \{w_t \leq \bar{\omega} p_{kt} A_t k_t\}$$

Upon default, a new manager arrives, defaults on the debt but keeps the capital.

Model - Equilibrium

An equilibrium for this economy is given by processes $\{p_{bt}, p_{kt}, C_{bt}\}_{t\geq 0}$, household decisions $\{c_t, k_{ht}, b_{ht}\}_{t\geq 0}$, and intermediary decisions $\{k_t, b_t, i_t\}_{t\geq 0}$ such that

1. given price processes, $\{c_t, k_{ht}, b_{ht}\}_{t\geq 0}$ solves household optimization problem
2. given price processes, $\{k_t, b_t, i_t\}_{t\geq 0}$ solves intermediary optimization problem
3. capital market clears: $K_t = k_t + k_{ht}$
4. risky bond market clears: $b_t = b_{ht}$
5. risk-free bond market clears: $w_{ht} - p_{kt} A_t k_{ht} - p_{bt} A_t b_{ht} = 0$
6. goods market clears: $c_t = A_t K_t - i_t A_t k_t$

Model Solution

The state of the economy is (θ_t, ω_t) where

$$\begin{aligned}\theta_t &= \frac{p_{kt} A_t k_t}{w_t} \Rightarrow \text{leverage} \\ \omega_t &= \frac{w_t}{w_t + w_{ht}} \Rightarrow \text{intermediary wealth share}\end{aligned}$$

Then, can obtain

$$dR_{kt} = \mu_{Rk,t} dt + \sigma_{ka,t} dZ_{at} + \sigma_{k\xi,t} dZ_{\xi t}$$

where $\{\mu_{Rk,t}, \sigma_{ka,t}, \sigma_{k\xi,t}\}$ are functions of (θ_t, ω_t).

Results - Financial Constraint

- The capital constraint reads

$$1 = \alpha^2 \theta_t^2 \left[\sigma_{ka,t}^2 + \sigma_{k\xi,t}^2 \right]$$

- higher volatility $\Rightarrow$ lower leverage
- higher α $\Rightarrow$ lower leverage

Results - Financial Constraint

Red line $= \Uparrow \alpha$ and **Dashed** line $= \Downarrow \omega$

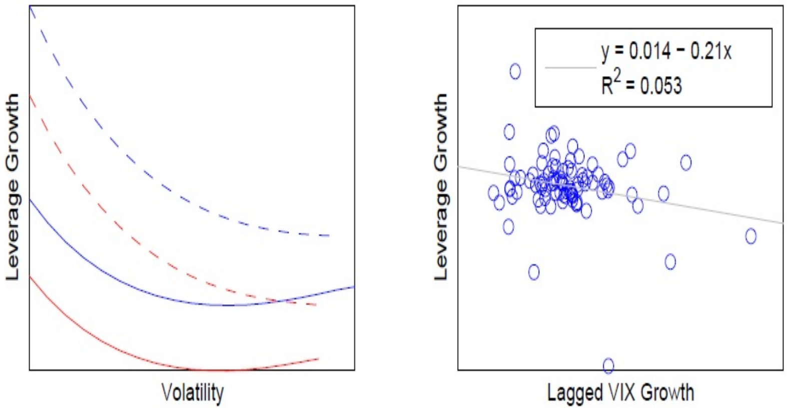

Figure: Volatility and Leverage

Results - Procyclical Leverage

Red line = $\Uparrow \alpha$ and **Dashed** line = $\Downarrow \omega$

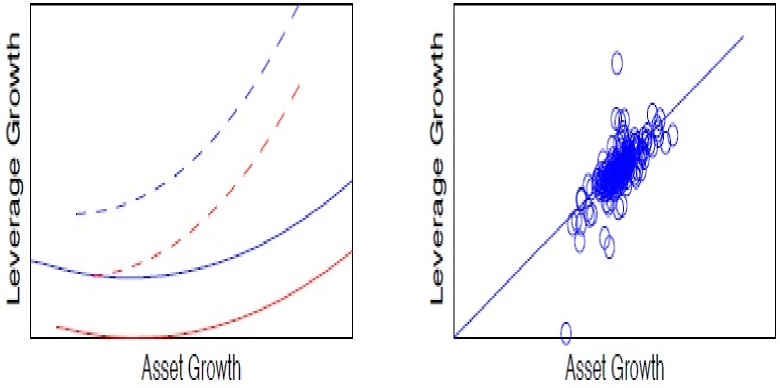

Figure: Procyclical Leverage

Results - Volatility Paradox

Recall

$$\tau_D = \inf_{t \geq 0} \{ w_t \leq \bar{\omega} p_{kt} A_t k_t \}$$

Define *distress probability* as

$$\delta_t(T) = \mathbb{P}(\tau_D \leq T | (\omega_t, \theta_t))$$

Results - Volatility Paradox

Red line = $\Uparrow \alpha$ and $T = 6$ months

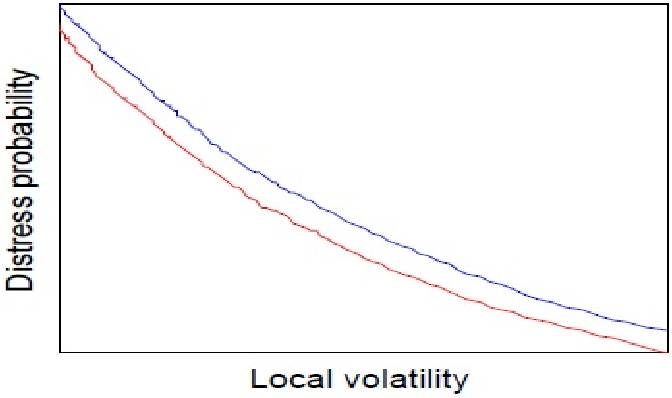

Figure: Volatility Paradox

Results - Macro-Prudential Policy

which is the optimal policy parameter α? Graph

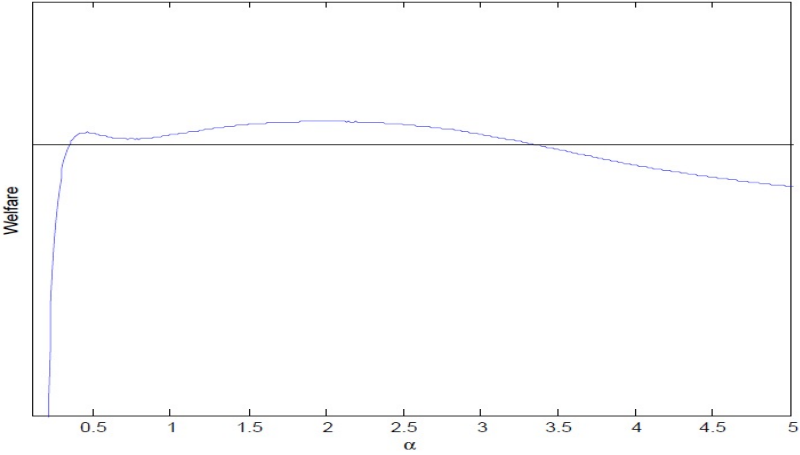

Figure: Household Welfare

Panza!

Motivation

Intermediary Procyclical Leverage - Adrian & Shin 2010

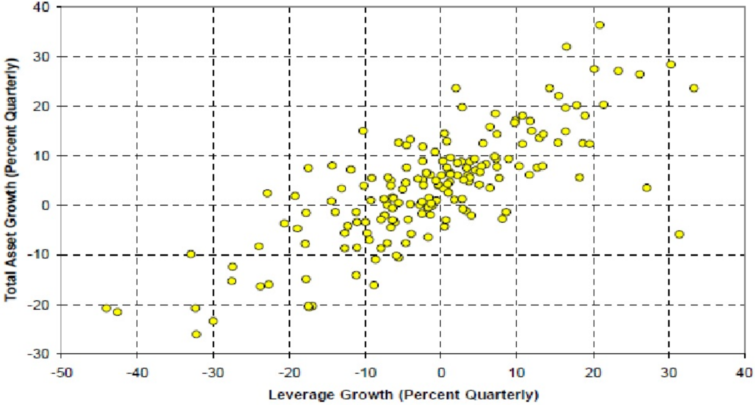

Figure: Procyclical financial intermediation

Return

Results Preview

1. The model can match many empirical facts
    1.1 procyclical leverage
    1.2 procyclical financial intermediation
    1.3 negative relation between leverage growth and assets excess returns
2. "volatility paradox": moments of low volatility are associated with foward-looking systemic risk
3. macro-prudential policy: there is an optimal amount of risk

Return

Mechanism: VaR constraint

vertical line $= -\frac{1}{\theta}$

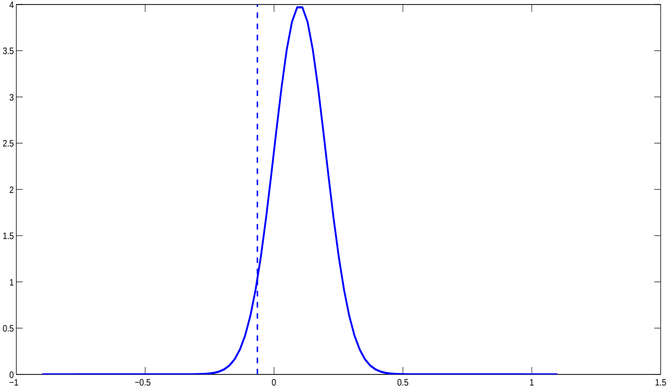

Figure: solution for θ

Mechanism: VaR constraint

$\Uparrow \sigma \rightarrow \Downarrow -\frac{1}{\theta} \rightarrow \Downarrow \theta$

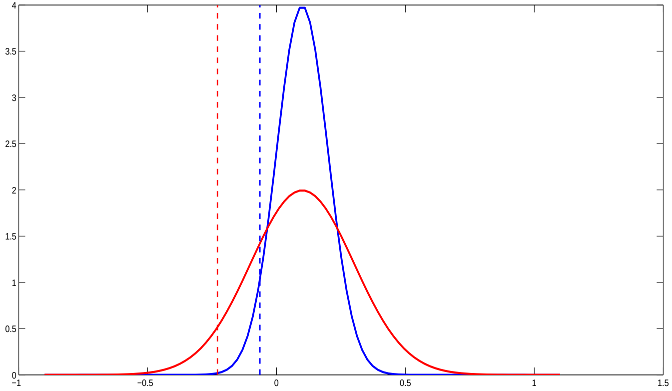

Figure: solution for θ

Return

Mechanism: VaR constraint

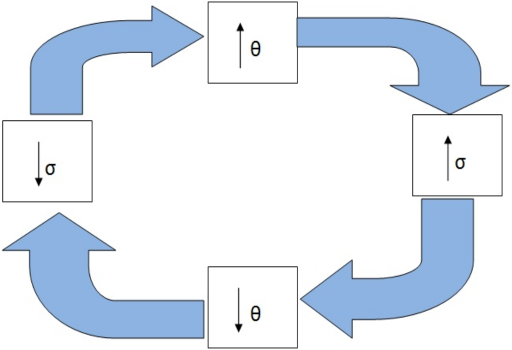

Figure: Mechanism

Model Solution

A solution of the model accounts to finding functions $\{\mu_{Rk,t}, \mu_{Rb,t}, \mu_{\omega,t}, \mu_{\theta,t}, \sigma_{Rk,t}, \sigma_{Rb,t}, \sigma_{\omega,t}, \sigma_{\theta,t}, r_{ft}\}$ as functions of the state (θ_t, ω_t) that satisfies our equilibrium definition.

Nice contribution of the paper:

this can be done by pencil and paper!

(not the case in He & Krishnamurthy neither Brunnemermeier & Sannikov)

Results - Macro-Prudential Policy

which is the optimal policy parameter α?

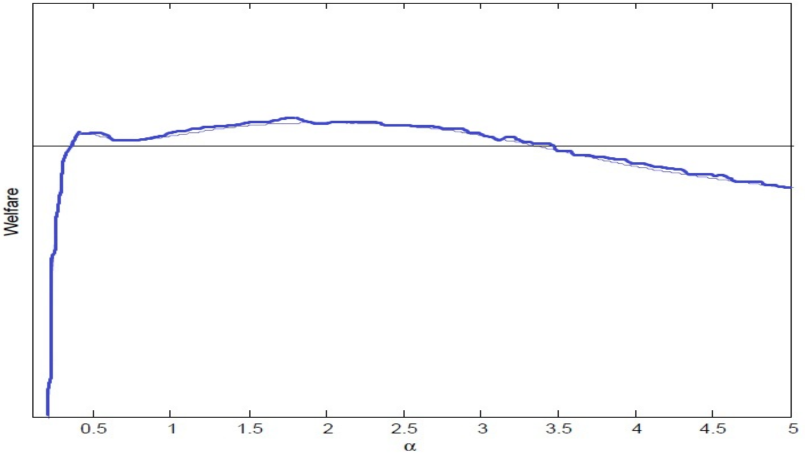

Figure: Household Welfare

Return