Pricing and Liquidity with Sticky Trading Plans

Bruno Biais, Johan Hombert, Pierre-Olivier Weill
Presented by Shengxing Zhang

November 13, 2012

environment

- continuous time
- Walrasian market for an asset
- asset supply: $s \in (0,1)$
- a continuum of investors, of total measure 1
- flow utility depends on preference type θ and asset holding q.
  $\theta \in \{l, h\}$, $q \in \mathbb{R}_+$, $v : \{l, h\} \times \mathbb{R}_+ \to \mathbb{R}$
- investors have deep pocket in cash

liquidity shocks and trading opportunities

- at $t = 0_-$, all investors hold s and have preference type h
- at $t = 0$, the preference type of all investors switches to l
- two idiosyncratic random processes during recovery
  - preference type recovers with Poisson rate γ
  - update trading plan with Poisson rate ρ

preference

$$
\begin{aligned}
v : \{h, l\} \times \mathbb{R}_+ &\rightarrow \mathbb{R}_+ \\
(\theta, q) &\mapsto v(\theta, q)
\end{aligned}
$$

$$
v(h, q) = \begin{cases} q, & \forall q \in [0, 1] \\ 1, & \forall q \in (1, \infty] \end{cases}
$$

$$
v(l, q) = \begin{cases} q - \delta q^{1+\sigma}/(1+\sigma), & \forall q \in [0, 1] \\ 1 - \delta/(1+\sigma), & \forall q \in (1, \infty] \end{cases}
$$

- $v_q(h, q) > v_q(l, q), \forall q \in (0, 1)$:
  l type will be the marginal investor on transition path
- $v(l, q)$ is strictly concave for $q \in [0, 1]$:
  asset holding for marginal investors to change smoothly
- $v_q(h, q) = v_q(l, q) = 0, \forall q \geq 1$:
  investors hold at most 1 unit in SS
- $v_q(h, 0) = v_q(l, 0) = 1$

definition of trading plan

trading plan updated at t is $\begin{array}{rcl} [t,\infty)\times\Omega & \to & \mathbb{R}_+ \\ (u,\omega) & \mapsto & q_{t,u}(\omega) \end{array}$

- u: the moment to purchase $q_{t,u}$

such that

- the stochastic process $(t,\omega)\mapsto q_{t,u}(\omega)$ is $\mathcal{F}_{t-}$ measurable, where $\{\mathcal{F}_t\}_{t\geq 0}$ is the filtration generated by the history of trading opportunities and preference shocks

the information at t can be reduced to: liquidity state θ_t

additional assumption

- the function $u \mapsto q_{t,u}(\omega)$ has bounded variations
- market price $p : t \mapsto p_t$ is continuously differentiable in the transition dynamics

investor's problem

$$\max_{q_{t,u}\geq 0, \forall u \geq t} \int_t^\infty e^{-r(u-t)} e^{-\rho(u-t)} \left\{ \mathbb{E}_t \left[v(\theta_u, q_{t,u}) \right] du - p_t dq_{t,u} \right\}$$

- $e^{-r(u-t)}$: discounting flow utility at u
- $e^{-\rho(u-t)}$: probability that the plan survives up until u
- $\mathbb{E}_t \left[v(\theta_u, q_{t,u}) \right]$: expectation conditional on information at t

Integration by part leads to

$$\max_{q_{t,u}\geq 0, \forall u \geq t} \int_t^\infty e^{-r(u-t)} e^{-\rho(u-t)} \left\{ \mathbb{E}_t \left[v(\theta_u, q_{tu}) \right] - \xi_u q_{t,u} \right\} du$$

where $\xi_u = r p_u - \dot{p}_u$
by point-wise optimization

$$\boxed{\mathbb{E}_t \left[v_q(\theta_u, q_{t,u}) \right] = \xi_u}$$

market clearing condition at u

$$s = \begin{array}{l} \int_0^u \rho e^{-\rho(u-t)} \left\{ (1-\mu_{ht}) q_{t,u}(\{\theta_t = l\}) + \mu_{ht} q_{t,u}(\{\theta_t = h\}) \right\} dt \\ + e^{-\rho u} s \end{array}$$

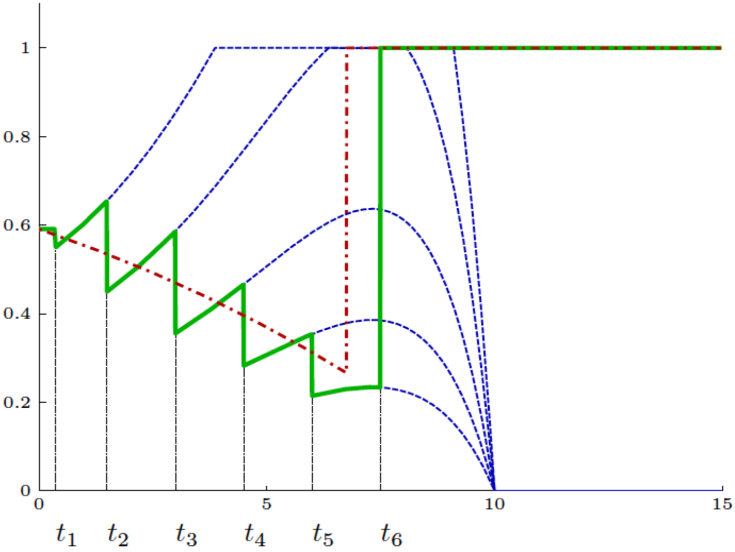
- $q_{t,u}(\{\theta_t = \theta\})$: trading plans made at t
- $\rho e^{-\rho(u-t)} dt$: measure of investors with trading plan $q_{t,\cdot}(\theta_t)$
- $\mu_{ht} = 1 - e^{\gamma t}$: fraction of h type at t

equilibrium

The equilibrium is a price path $p : t \mapsto p_t$, and a collection of trading plans $\{q_{t,\cdot}(\theta_t) : u \mapsto q_{t,u}(\theta_t)\}_{\forall t \geq 0, \theta \in l,h}$ such that

- given the price path, trading plan $q_{t,\cdot}(\theta_t)$ solves the problem of investors of θ_t updating plan at t
- market clears at any t

trading dynamics



characterization

$$\boxed{\mathbb{E}_t\left[v_q(\theta_u, q)\right] = \xi_u}$$

- $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\} = q - \delta e^{-\gamma(u-t)} q^\sigma,\ \forall q \in [0, 1]$
  where $e^{-\gamma(u-t)}$ is the probability to remain low type
- $v_q(\theta_u = l, q) = 1 - \delta q^\sigma,\ \forall q \in [0, 1]$
- $v_q(\theta_u = h, q) = 1,\ \forall q \in [0, 1]$

characterization

$$\boxed{\mathbb{E}_t\left[v_q(\theta_u, q)\right] = \xi_u}$$

- $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\} = q - \delta e^{-\gamma(u-t)} q^\sigma,\ \forall q \in [0, 1]$
  where $e^{-\gamma(u-t)}$ is the probability to remain low type
- $v_q(\theta_u = l, q) = 1 - \delta q^\sigma,\ \forall q \in [0, 1]$
- $v_q(\theta_u = h, q) = 1,\ \forall q \in [0, 1]$

$\Rightarrow$ $v'(\theta_u = h, q) = 1 > \mathbb{E}_t\{v'(\theta_u, q)|\theta_t = l\} > v'(\theta_u = l, q)$

characterization

$$\boxed{\mathbb{E}_t\left[v_q(\theta_u, q)\right] = \xi_u}$$

- $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\} = q - \delta e^{-\gamma(u-t)} q^\sigma, \; \forall q \in [0,1]$
  where $e^{-\gamma(u-t)}$ is the probability to remain low type
- $v_q(\theta_u = l, q) = 1 - \delta q^\sigma, \; \forall q \in [0,1]$
- $v_q(\theta_u = h, q) = 1, \; \forall q \in [0,1]$

$\Rightarrow$ $v'(\theta_u = h, q) = 1 > \mathbb{E}_t\{v'(\theta_u, q)|\theta_t = l\} > v'(\theta_u = l, q)$

$\Rightarrow$ $q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l)$

characterization

$$\boxed{\mathbb{E}_t\left[v_q(\theta_u, q)\right] = \xi_u}$$

- $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\} = q - \delta e^{-\gamma(u-t)} q^\sigma, \; \forall q \in [0,1]$ where $e^{-\gamma(u-t)}$ is the probability to remain low type
- $v_q(\theta_u = l, q) = 1 - \delta q^\sigma, \; \forall q \in [0,1]$
- $v_q(\theta_u = h, q) = 1, \; \forall q \in [0,1]$

$\Rightarrow$ $v'(\theta_u = h, q) = 1 > \mathbb{E}_t\{v'(\theta_u, q)|\theta_t = l\} > v'(\theta_u = l, q)$

$\Rightarrow$ $q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l)$

- $q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l)$ and $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\}$ increasing in u: *round-trip trading*

characterization

$$\boxed{\mathbb{E}_t\left[v_q(\theta_u, q)\right] = \xi_u}$$

- $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\} = q - \delta e^{-\gamma(u-t)} q^{\sigma}, \forall q \in [0,1]$
  where $e^{-\gamma(u-t)}$ is the probability to remain low type
- $v_q(\theta_u = l, q) = 1 - \delta q^{\sigma}, \forall q \in [0,1]$
- $v_q(\theta_u = h, q) = 1, \forall q \in [0,1]$

$\Rightarrow$ $v'(\theta_u = h, q) = 1 > \mathbb{E}_t\{v'(\theta_u, q)|\theta_t = l\} > v'(\theta_u = l, q)$

$\Rightarrow$ $q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l)$

- $q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l)$ and $\mathbb{E}_t\{v_q(\theta_u, q)|\theta_t = l\}$ increasing in u: *round-trip trading*
- $q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l)$: *delay in recovery*

trade volume

- round-trip trades raise trade volume
- infrequent updating reduces trade volume
- as $\rho \to \infty$, trade volume may be strictly higher than that of continuous trading

delay in trade

- recovery takes longer
- welfare lower than that in continuous trading
- constrained efficient: endogenous intermediation by trading low type
- results depend on the curvature of utility of l type

price dynamics

- if $s < \sigma/(\sigma+1)$, prices are lower under sticky trading plans
- if otherwise, price may be hump-shaped and may be higher sometimes

instrument of implementation

- tools: limit order, market order, trigger buy/sell order
- trading plan implementable with a rich set of instrument
- if limited: allocation point-wise optimization may not be feasible