Pricing and Liquidity with Sticky Trading Plans

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- continuous time
- Walrasian market for an asset
- asset supply: \( s \in (0, 1) \)
- a continuum of investors, of total measure 1
- flow utility depends on preference type \( \theta \) and asset holding \( q \).
  \[ \theta \in \{l, h\}, \ q \in \mathbb{R}_+, \ v : \{l, h\} \times \mathbb{R}_+ \rightarrow \mathbb{R} \]
- investors have deep pocket in cash
liquidity shocks and trading opportunities

- at \( t = 0^- \), all investors hold \( s \) and have preference type \( h \)
- at \( t = 0 \), the preference type of all investors switches to \( l \)
- two idiosyncratic random processes during recovery
  - preference type recovers with Poisson rate \( \gamma \)
  - update trading plan with Poisson rate \( \rho \)
preference

\[ \nu : \{h, l\} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \]
\[ (\theta, q) \mapsto \nu(\theta, q) \]

\[ \nu(h, q) = \begin{cases} 
q, & \forall q \in [0, 1] \\
1, & \forall q \in (1, \infty] 
\end{cases} \]

\[ \nu(l, q) = \begin{cases} 
q - \delta q^{1+\sigma} / (1 + \sigma), & \forall q \in [0, 1] \\
1 - \delta / (1 + \sigma), & \forall q \in (1, \infty] 
\end{cases} \]

- \( \nu_q(h, q) > \nu_q(l, q), \forall q \in (0, 1) \): 
  l type will be the marginal investor on transition path

- \( \nu(l, q) \) is strictly concave for \( q \in [0, 1] \): 
  asset holding for marginal investors to change smoothly

- \( \nu_q(h, q) = \nu_q(l, q) = 0, \forall q \geq 1 \): 
  investors hold at most 1 unit in SS

- \( \nu_q(h, 0) = \nu_q(l, 0) = 1 \)
definition of trading plan

trading plan updated at $t$ is $\left[ t, \infty \right) \times \Omega \rightarrow \mathbb{R}_+$

$$(u, \omega) \mapsto q_{t,u}(\omega)$$

- $u$: the moment to purchase $q_{t,u}$ such that

  - the stochastic process $(t, \omega) \mapsto q_{t,u}(\omega)$ is $\mathcal{F}_t$ measurable, where $\{\mathcal{F}_t\}_{t \geq 0}$ is the filtration generated by the history of trading opportunities and preference shocks

the information at $t$ can be reduced to: liquidity state $\theta_t$
additional assumption

- the function $u \mapsto q_{t,u}(\omega)$ has bounded variations
- market price $p : t \mapsto p_t$ is continuously differentiable in the transition dynamics
investor’s problem

\[
\max_{q_t, u \geq 0, \forall u \geq t} \int_t^\infty e^{-r(u-t)} e^{-\rho(u-t)} \left\{ \mathbb{E}_t [v(\theta_u, q_{t,u})] \, du - p_t \, dq_{t,u} \right\}
\]

- \( e^{-r(u-t)} \): discounting flow utility at \( u \)
- \( e^{-\rho(u-t)} \): probability that the plan survives up until \( u \)
- \( \mathbb{E}_t [v(\theta_u, q_{t,u})] \): expectation conditional on information at \( t \)

Integration by part leads to

\[
\max_{q_t, u \geq 0, \forall u \geq t} \int_t^\infty e^{-r(u-t)} e^{-\rho(u-t)} \left\{ \mathbb{E}_t [v(\theta_u, q_{tu})] - \xi_u \, q_{t,u} \right\} \, du
\]

where \( \xi_u = r p_u - \dot{p}_u \)

by point-wise optimization

\[
\mathbb{E}_t [v_q(\theta_u, q_{t,u})] = \xi_u
\]
market clearing condition at $u$

$$s = \int_0^u \rho e^{-\rho(u-t)} \left\{ (1 - \mu_{ht}) q_{t,u}(\{\theta_t = l\}) + \mu_{ht} q_{t,u}(\{\theta_t = h\}) \right\} dt$$

- $q_{t,u}(\{\theta_t = \theta\})$: trading plans made at $t$
- $\rho e^{-\rho(u-t)} dt$: measure of investors with trading plan $q_{t,\theta}(\theta_t)$
- $\mu_{ht} = 1 - e^{\gamma t}$: fraction of $h$ type at $t$
The equilibrium is a price path $p : t \mapsto p_t$, and a collection of trading plans $\{ q_{t,.}(\theta_t) : u \mapsto q_{t,u}(\theta_t) \}_{\forall t \geq 0, \theta \in I, h}$ such that

- given the price path, trading plan $q_{t,.}(\theta_t)$ solves the problem of investors of $\theta_t$ updating plan at $t$
- market clears at any $t$
trading dynamics
\[ \mathbb{E}_t [v_q(\theta_u, q)] = \xi_u \]

- \( \mathbb{E}_t \{ v_q(\theta_u, q) | \theta_t = l \} = q - \delta e^{-\gamma(u-t)} q^\sigma, \forall q \in [0,1] \)
  where \( e^{-\gamma(u-t)} \) is the probability to remain low type
- \( v_q(\theta_u = l, q) = 1 - \delta q^\sigma, \forall q \in [0,1] \)
- \( v_q(\theta_u = h, q) = 1, \forall q \in [0,1] \)
characterization

\[ \mathbb{E}_t [\nu_q(\theta_u, q)] = \xi_u \]

- \( \mathbb{E}_t \{ \nu_q(\theta_u, q) | \theta_t = l \} = q - \delta e^{-\gamma(u-t)} q^\sigma, \ \forall q \in [0, 1] \) where \( e^{-\gamma(u-t)} \) is the probability to remain low type
- \( \nu_q(\theta_u = l, q) = 1 - \delta q^\sigma, \ \forall q \in [0, 1] \)
- \( \nu_q(\theta_u = h, q) = 1, \ \forall q \in [0, 1] \)

\( \Rightarrow \) \( \nu'(\theta_u = h, q) = 1 > \mathbb{E}_t \{ \nu'(\theta_u, q) | \theta_t = l \} > \nu'(\theta_u = l, q) \)
\[ E_t [v_q(\theta_u, q)] = \xi_u \]

- \[ E_t\{v_q(\theta_u, q)|\theta_t = l\} = q - \delta e^{-\gamma(u-t)}q^\sigma, \ \forall q \in [0, 1] \]
  where \( e^{-\gamma(u-t)} \) is the probability to remain low type

- \[ v_q(\theta_u = l, q) = 1 - \delta q^\sigma, \ \forall q \in [0, 1] \]

- \[ v_q(\theta_u = h, q) = 1, \ \forall q \in [0, 1] \]

\[ v'(\theta_u = h, q) = 1 > E_t\{v'(\theta_u, q)|\theta_t = l\} > v'(\theta_u = l, q) \]

\[ q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l) \]
characterization

\[ E_t [v_q(\theta_u, q)] = \xi_u \]

- \( E_t \{ v_q(\theta_u, q) | \theta_t = l \} = q - \delta e^{-\gamma(u-t)} q^\sigma \), \( \forall q \in [0, 1] \)
  where \( e^{-\gamma(u-t)} \) is the probability to remain low type
- \( v_q(\theta_u = l, q) = 1 - \delta q^\sigma \), \( \forall q \in [0, 1] \)
- \( v_q(\theta_u = h, q) = 1 \), \( \forall q \in [0, 1] \)

\[ \Rightarrow v'(\theta_u = h, q) = 1 > E_t \{ v'(\theta_u, q) | \theta_t = l \} > v'(\theta_u = l, q) \]

\[ \Rightarrow q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l) \]
- \( q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l) \) and \( E_t \{ v_q(\theta_u, q) | \theta_t = l \} \)
  increasing in \( u \): *round-trip trading*
characterization

\[ \mathbb{E}_t [v_q(\theta_u, q)] = \xi_u \]

- \[ \mathbb{E}_t \{ v_q(\theta_u, q) | \theta_t = l \} = q - \delta e^{-\gamma(u-t)} q^\sigma, \quad \forall q \in [0, 1] \]
where \( e^{-\gamma(u-t)} \) is the probability to remain low type
- \[ v_q(\theta_u = l, q) = 1 - \delta q^\sigma, \quad \forall q \in [0, 1] \]
- \[ v_q(\theta_u = h, q) = 1, \quad \forall q \in [0, 1] \]
\[ \Rightarrow v'(\theta_u = h, q) = 1 > \mathbb{E}_t \{ v'(\theta_u, q) | \theta_t = l \} > v'(\theta_u = l, q) \]
\[ \Rightarrow q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l) \]
- \[ q_{t,u}(\theta_t = l) > q_{u,u}(\theta_u = l) \]
and \( \mathbb{E}_t \{ v_q(\theta_u, q) | \theta_t = l \} \) increasing in \( u \): *round-trip trading*
- \[ q_{u,u}(\theta_u = h) > q_{t,u}(\theta_t = l) : \text{delay in recovery} \]
trade volume

- round-trip trades raise trade volume
- infrequent updating reduces trade volume
- as $\rho \to \infty$, trade volume may be strictly higher than that of continuous trading
delay in trade

- recovery takes longer
- welfare lower than that in continuous trading
- constrained efficient: endogenous intermediation by trading low type
- results depend on the curvature of utility of $l$ type
price dynamics

- if $s < \sigma/(\sigma + 1)$, prices are lower under sticky trading plans
- if otherwise, price may be hump-shaped and may be higher sometimes
instrument of implementation

- tools: limit order, market order, trigger buy/sell order
- trading plan implementable with a rich set of instrument
- if limited: allocation point-wise optimization may not be feasible